

William Feller: A Biography

by *René L. Schilling* from Dresden and *Wojbor A. Woyczyński* from Cleveland

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William Feller was born on 7 July 1906 in Zagreb, Croatia, which was then a Southern province of the Austro-Hungarian Empire. He was the ninth of twelve children of Eugen Viktor Feller and Ida Feller. Following the Roman-Catholic tradition, the young baby boy was named Vilibald after the saint (St. Willibald) whose feast day fell on his birthday. In the church register of births (city register)¹ he had been registered as Vilibald Srećko² Feller. Feller called himself Vilim³ (which is the Croatian form of William), and throughout his life he would adopt the native versions of his first name depending on the country he lived in: thus in Germany and Scandinavia he called himself Willi/Willy, and William/Will in America.

Feller's father Eugen was born in 1871 in Lemberg, Galicia, in the Northeastern corner of the Austro-Hungarian Empire (now Lviv, in Western Ukraine), and died in 1936 in Zagreb. His wife Ida was born in 1870 (place unknown), and passed away in Zagreb two years after her husband's death. His paternal grandparents were David Feller and Elizabeth (Elsa) Holzer from Lemberg, and his maternal grandparents were Ferdinand Oemichen and Hermina Peerz (or Perc). Feller's parents owned a pharmaceutical company. The then-famous, now-obscure Elsa fluid, named after Eugen's mother, was one of the mainstays of the enterprise. It had been marketed as an elixir capable to cure all kinds of maladies, such as headaches, colds, back pains, etc., and was the source of the family's substantial wealth. Like many upper-class families, the Fellers were bilingual and William spoke both Croatian and German. Young William was raised in the Roman-Catholic tradition even though his paternal grandfather David was Jewish; he had converted to Catholicism when marrying Elsa.⁴ Feller's ancestors probably came to Zagreb during the second half of the nineteenth century. More details on Feller's parents and grandparents can be found in the two studies by Fatović-Ferenčić, and Ferber-Bogdan [10, 11].

Zagreb 1906–1925

We do not know the exact year when the Fellers moved to their modern, spacious and elegantly decorated villa on 31a, Jurjevska street,⁵ but it is safe to assume that William spent most of his childhood there. Judging by family photos and personal stories, they lived in a pleasant and productive environment. Several of William's brothers had successful careers of their own: Ferdinand (Ferdo) (1897–1960) was a well-known pharmacist, Miroslav (Fritz) (1901–1961) was a poet, novelist, philosopher, and an art critic, and Marijan (1903–1974) was a pianist with an academic career in Zagreb and Sarajevo.⁶

¹Žubričić [47, p. 9]. The references [Feller 19nn] and [*Feller 19nn] (the star indicating that the respective paper is not contained in these Selecta) refer to Feller's bibliography, while [n] points to the list of references at the end of this essay.

²Srećko is the Croatian equivalent of Felix, 'the lucky'.

³The rather anecdotal story in Seneta [40, p. 87] is based on the claim that July 7 is St. Willibald in the "(German) Catholic saint's list" while it is Sv. Vilim's feast day in the "Croatian Catholic list" which cannot be verified (www.namecalendar.net, accessed 3 August 2014, gives for Vilim the dates 6 April, 23 May, 8 June and 25 June, the Croatian Catholic Calendar www.hkr.hr/kalendar, accessed 29 August 2014, gives the dates 10 February, 23 May, 8 June and 29 July). It is more likely that Vili(m) was used as a family nickname instead of the more formal Vilibald; also in German, it is common practice to abbreviate Willibald by Willi or Willy.

⁴Žubričić [47, pp. 2 ff.]

⁵The Villa was built in 1910/11 and designed by Mathias Feller from Munich, William's uncle. The whole September 1914 issue of the German architectural magazine *Innendekoration* [12] is devoted to its description.

⁶Žubričić [47, pp. 26 ff.] and the Croatian Biographical Encyclopedia [22].



Picture 1: William Feller as a young boy, aged 4. The handwritten note in the lower left corner says “Unser Willy” (our Willy). The photograph is taken from the family’s picture album, courtesy of Joanne Elliott.

Not much is known about William Feller’s schooldays. Until the end of the first World War (1918), Croatia and Zagreb (Agram in German) were part of the autonomous “Kingdom of Dalmatia, Croatia and Slavonia”, associated with the Kingdom of Hungary, itself part of the Austro-Hungarian monarchy. From 1918 onwards it was part of the Kingdom of Serbs, Croats and Slovenes, which was renamed “Kingdom of Yugoslavia” in 1929. In Feller’s days the educational system still followed the Austrian and German tradition, and William (as did five of his brothers) attended the *I. Realgymnasium*^{7,8} in Zagreb, graduating in June 1923.⁹ During his first years there he was a private pupil¹⁰; he also skipped two years.¹¹ One of Feller’s private teachers was Stanko Vlögel, a mathematician and faculty member at the University of Zagreb.¹²

In October 1923 Feller enrolled at the University of Zagreb as a student of Mathematics and Physics. From the matriculation forms, which for the summer terms of 1924, and 1925, and the winter term of 1924/25, are reprinted in Žubrinić [47, p. 14, p. 23], we learn that Feller was then using Vilim (rather than Vilibald) as his first name. The forms also contain information about the courses Feller had been taking: Mathematics I–IV (with M. Kiseljak), Differential and Integral Calculus (V. Varićak), Infinite Series (S. Bohniček), Number Theory 1,2 (S. Bohniček), Theory of Real Functions (V. Vranić), Calculus of Variations (V. Varićak), and two mathematical seminars (V. Varićak), along with a panoply of experimental and theoretical physics¹³, some chemistry, and

⁷German for a “secondary school that prepares students for the university, that offers Latin but no Greek, and that typically emphasizes sciences and modern languages” (www.merriam-webster.com/dictionary/realgymnasium, accessed 30/7/2014).

⁸The former I. Realgymnasium today houses the Mimara museum of art, one of Croatia’s foremost art collections.

⁹Žubrinić [47, p. 12]

¹⁰I.e. he had private teachers at home, only the exams to progress to the next year were taken at the Realgymnasium.

¹¹ “[I]ch [...] durfte mit Erlaubnis der Regierung zwei Klassen kontrahieren” [1, handwritten CV in the Promotionsakte].

¹²Žubrinić [47, p. 12]

¹³Among the courses were: Mechanics & Acoustics, Experiments in Physics, Theoretical Mechanics, Thermodynamics, Optics, Theory of Heat and a Physics Colloquium.



Picture 2: The Fellers' Villa at 31a, Jurjevska Street, in Zagreb. Clockwise from the top left: View from the South in 1913/14 and today, from the East (today), and the West (in 1913/14). Photographs on the left side courtesy of Joanne Elliott, on the right side courtesy of Zoran Vondraček.

a sample of courses in pedagogy and psychology. The fact that Feller simultaneously studied two subjects was well within the Austro-German tradition. The final exam was either the *Staatsexamen*, which would qualify him to teach at grammar schools (*Gymnasium* and *Realgymnasium*), or the doctorate (the *Diplom*, Master's Diploma, was introduced only later, in the 1930s – 50s). From today's perspective, Feller's educational pathway at the University of Zagreb is comparable to the standard education at an average German or Austrian university of the 1920s. Among Feller's teachers Vladimir Varićak (1865–1942) was the most internationally visible scientist, renowned for his contributions to Einstein's theory of relativity and non-Euclidean geometry. Very likely Varićak's lectures and mathematical seminars made a lasting impact on Feller's early work in geometry.

The German years: Göttingen & Kiel 1925–1933

In Constance Reid's biographies of David Hilbert and Richard Courant we find a confirmation of the fact that

At the beginning of the twentieth century, mathematics students all over the world were receiving the same advice: "Pack your suitcase and take yourself to Göttingen!"¹⁴

¹⁴Reid [33, p. 102]

Thus, after four semesters of study at Zagreb,¹⁵ Feller enrolled for the winter term of 1925/26 at the University of Göttingen. After the first World War,

... in Göttingen there was again a scientific paradise. Gifted students flocked to the university. There was a constant procession of distinguished visitors from all over the world. Sometimes they came merely for a single talk or a series of talks to the *mathematische Gesellschaft*. Often they lectured for a full term as guest professors. The very air seemed to [...] crackle [...] with scientific electricity.¹⁶

Feller was only nineteen when he moved to Göttingen. We do not know many details of his student life, but the *Promotionsakte* (dissertation file) [1] contains his address: Nikolausbergerweg 43, Göttingen. He attended the *Anfängerpraktikum*, problem classes for beginners, a revolutionary idea introduced by Richard Courant¹⁷, and very quickly caught the attention of Courant's assistants.¹⁸ Reid [34, p. 111] contains an anecdote about the first contact between Feller and Courant:

At the beginning of the [winter] term in 1925, [...] [the assistants] promptly alerted Courant to the presence of Willy Feller. After the third calculus lecture, to Feller's amazement, the professor – an unbelievably august personage to a European student of that day – approached. Questioning the boy about his education in his native land, Courant discovered that Feller was already doing mathematics on his own. He told him to bring his work to the next lecture. Even thus instructed, Feller was too bashful to produce his papers on the appointed day. The next morning he was awakened by a commotion on the stairs leading to his attic room. There was a knock on the door. Courant entered and left a few moments later with the desired papers.¹⁹

Very likely the papers were related to the topics covered in the Mathematics Seminar Feller attended in his Zagreb days. Soon thereafter Feller joined Courant's circle of students and collaborators, which proved to be hugely beneficial to him: on 10 August 1926, after only two semesters at Göttingen, Feller, who had just turned twenty, submitted his Ph.D. thesis *Über algebraisch rektifizierbare transzendente Kurven* [*Feller 1926]. Courant gave the thesis the grade of I, or at least II,²⁰ remarking

that [Feller] independently, without outside input, found the topic and worked on it, notably in an environment, (Agram), where no outside input was possible.²¹

Courant was slightly unhappy about the fact that the topic was not completely in the mainstream of the mathematics of the day,²² but he found this shortcoming balanced by the complete independence of Feller's effort. Indeed, the topic of Feller's thesis (algebraically rectifiable transcendental curves) was much closer to Varićak's work than to anything done at Göttingen at that time, and Feller would never come back to this line of research.

The 90-minute oral exam took place on 3 November 1926 and the examination subjects were Physics (examiner: J. Franck), Mathematical Analysis (G. Herglotz), and Mathematics – Geometry (R. Courant). Feller passed this part with a I (outstanding). German universities require that the thesis be printed, so the degree was not finally awarded until 18 July 1927, after the thesis had been accepted for publication in *Mathematische Zeitschrift* [Feller 1928].

¹⁵Feller did not obtain a formal degree from Zagreb. The *Promotionsakte* from Göttingen [1] lists an *Abgangszeugnis* (transcript of marks) of the University in (*sic!*) Agram (Zagreb) dated 13 October 1925. The *Abgangszeugnis*, however, was returned to the candidate after the defence of the thesis and is not part of the *Promotionsakte*.

¹⁶Reid [34, p. 119]

¹⁷Reid [34, p. 99]. In those years, Courant's assistants, O. Neugebauer and K. O. Friedrichs were in charge of the *Anfängerpraktikum*. Feller would meet them again later in his career.

¹⁸Around the same time quite a few very gifted students were at Göttingen, e.g. Herbert Busemann (1905–1994), Hans Lewy (1904–1988), John von Neumann (1903–1957), and Franz Rellich (1906–1955).

¹⁹Rota also mentions this as an example of one of Feller's "bombastic stories" [38, p. 230].

²⁰*Promotionsakte* [1]. At German universities (and also at Göttingen) a PhD thesis/examination is marked on the following scale: I is *outstanding, summa cum laude*, II *very good, magna cum laude*, III *good, cum laude*, IV *satisfactory, rite*.

²¹dass er [Feller] selbständig, ohne jede äussere Anregung, sein Problem sich gestellt und bearbeitet hat und zwar in einer Umgebung, (Agram), in welcher er keinerlei Anregung von aussen empfangen konnte.

²²Courant notes "that the choice of the topic leads away from those questions which nowadays appear to be interesting and important" ["dass die Stellung des Themas etwas aus dem Rahmen der Fragen hin[aus]führt, die uns heute interessant und wichtig erscheinen"].



Picture 3: William Feller in his twenties or thirties. Photographs courtesy of Joanne Elliott.

One year later Abraham Adolf Fraenkel²³ invited the “highly gifted” Feller to join Kiel University. He moved there in 1928 and almost immediately (in February of 1929), with Fraenkel’s backing, obtained the habilitation in mathematics,²⁴ and became *Privatdozent*. The habilitation thesis [*Feller 1929] (published as [Feller 1930]) *Über die Lösungen der linearen partiellen Differentialgleichungen zweiter Ordnung vom elliptischen Typus*²⁵ was obviously influenced by his time at Göttingen (see Jacob’s detailed commentary [24] in these Selecta).

At about the same time, in October 1929, Erhard Tornier started his job as a visiting professor at Kiel,²⁶ an innocuous event at the time which had momentous repercussions in Feller’s life four years later. He became Feller’s colleague and it is very likely that they began discussing mathematics right away since Tornier was interested in the foundational questions of probability theory. They were then a very active topic in mathematics and several big open problems remained following introduction of von Mises’ Kollektivs (1919), and the appearance of Steinhaus’ (1923) *Fundamenta Mathematicae* paper [44] which tried to provide probability theory with a measure-theoretic foundation in the sense of Lebesgue. But Kolmogorov’s axiomatization (1933) had not yet been published, and Tornier’s strategy could be described as fitting in-between the above approaches.²⁷ This may have been Feller’s first contact with modern probability and measure theory. Having completed his visiting position appointment at Kiel, Tornier left for Halle but at the end of 1931 he asked Fraenkel

to be allowed to transfer his habilitation [which he obtained at Halle in 1930] to Kiel, in order to be ‘in my [Fraenkel’s] environment and to gain substantial knowledge from me’. I knew Tornier only superficially,²⁸ but I did grant his wish, *mainly because of Feller* [our emphasis].²⁹

²³of Zermelo–Fraenkel fame, worked on number theory, set theory and the foundations of mathematics. He came to Kiel in 1928. His relation to Feller and Tornier is described in his autobiography *Lebenskreise* [17, pp. 154 f].

²⁴Habilitation is a post-doctoral degree that bestows on the recipient the *venia legendi*, the permission to teach, examine and supervise students independently at the university which conferred the degree. Usually a full professor of the faculty has to support one’s application for habilitation.

²⁵On the solutions of second-order linear partial differential equations of elliptic type

²⁶Hochkirchen [21, p. 26]

²⁷Hochkirchen [21]. Feller’s paper [*Feller 1938a] is largely trying to explain why Tornier’s theory is in line with Kolmogorov’s approach; cf. also Fischer [15] in these Selecta.

²⁸When Tornier was a visiting faculty member at Kiel, Fraenkel was in Jerusalem. Both, however, have had the same doctoral advisor: Kurt Hensel at the University of Marburg.

²⁹Fraenkel [17, p. 131; our translation from German]

Thus Feller changed his subject yet again, and started a collaboration with Erhard Tornier on measure theory and the foundations of probability theory, (see the essay of Fischer [15] in these *Selecta*) which resulted in several joint publications [*Feller 1931, *Feller 1932a, *Feller 1932b] and his own papers on Lebesgue integration [*Feller 1932c] appear; the later papers [Feller 1934a] (with Busemann), [*Feller 1938a], and [Feller 1939c], also belong to this circle of ideas.

The other side of the coin was that Tornier turned out to be an ardent Nazi supporter joining the Nazi party (NSDAP) as early as 1932.³⁰ As soon as Hitler came to power on 30 January 1933, Tornier started denouncing his ‘Jewish’ and non-Aryan colleagues, including Feller, whose paternal grandfather was Jewish. In Nazi language, Feller was a *Vierteljude* (‘Quarter-Jew’) and, as such, not fit for civil service. The *Gesetz zur Wiederherstellung des Berufsbeamtentums* (BBG)³¹ required all staff members to fill out a questionnaire and to prove their Aryan ancestry (*Ariernachweis*) – and so did Feller.³² Since Feller was considered to be a *Vierteljude*, the *Freie Kieler Studentenschaft* – a Nazi student organization – demanded on 21 April 1933 (two days after Feller’s questionnaire had been evaluated), that Feller be suspended, and this request was supported by Tornier who emphasized Feller’s Croatian, hence Slavic, origins. On the basis of the BBG, Feller’s right to teach at Kiel university was revoked on 9 September 1933.³³ A month later he left Germany for Denmark.³⁴

During Feller’s Kiel days a young student, Clara Nielsen, attended one of his classes.³⁵ Several years later, in 1938, she became his wife. Unfortunately, we do not have much information regarding Clara Nielsen. Most likely she was German, possibly a member of the Danish minority in Schleswig; we know from Feller’s letter³⁶ that “Clara’s father (who is 2/3 danish [*sic!*], her mother being dutch [*sic!*]) lives in Flensburg”, a city on the German–Danish border. It seems she was born on 15 September 1910,³⁷ which would have made her twenty-four years old in 1933, the year Feller left Germany.

European Exile: Copenhagen & Stockholm 1933–1939

Feller spent the Academic Year 1933/34 in Copenhagen, where Harald and Niels Bohr had been actively helping German emigrés. There he ran again into Herbert Busemann and Otto Neugebauer who, in the years 1934–38, continued editing the *Zentralblatt für Mathematik und ihre Grenzgebiete*³⁸ from Denmark. Feller knew both from his Göttingen days and very quickly started

³⁰He joined on 1 May 1932, after the philosophical faculty at Kiel agreed to recognize his Halle habilitation on 2 March 1932, cf. [21, p. 29].

³¹Act for the re-instatement of the professional civil service (7 April 1933) a rather euphemistically named Nazi law which allowed to cleanse the civil service from non-Aryan and politically non-conforming members. Its §3 is one of the earliest Aryan paragraphs. The full text is available under www.documentarchiv.de/ns/beamtenges.html (accessed 1 August 2014), excerpts are in Uhlig [45, pp. 143 ff].

³²Doob [9] writes of an “Nazi oath” which Feller “refused to sign [...] and was forced to leave”; this story is reiterated at several places but it cannot be confirmed. On the contrary, Feller filled out all required forms and he was forced to leave on the basis of his ancestral information. The form, which appeared as an appendix to the BBG, is available online alex.onb.ac.at/cgi-content/alex?apm=0&aid=dra&datum=19330004&zoom=2&seite=00000253&ues=0&x=13&y=7 (accessed 20 August 2014).

³³See the official web-page of the University of Kiel, www.uni-kiel.de/ns-zeit/bios/feller-willy.shtml (accessed 31 July 2014) and the documentation in Uhlig [45, p. 23].

³⁴Uhlig [45, p. 23]

³⁵Doob [9, p. xvii], Birnbaum [4, p. iv]

³⁶Handwritten letter of Feller to Borge Jessen [13]

³⁷U.S. Social Security Death Index, accessed on 1 August 2014 via death-records.findthebest.com/1/107145366/Clara-Feller. This record shows that a Clara Feller from Princeton, Mercer County, was born on 15 September 1910 and died on 1 October 1973. Her Social Security number was also provided.

³⁸The following historical remarks on the *Zentralblatt* are from zbmath.org/about/ (accessed 1 August 2014): “The *Zentralblatt für Mathematik und ihre Grenzgebiete* was founded in 1931 with the aim to publish reviews of the entire world literature in mathematics and related areas. *Zentralblatt* became the second comprehensive review journal for mathematics in Germany after the *Jahrbuch über die Fortschritte der Mathematik* (established in 1868) which has been active until the 1940s. Although the *Zentralblatt* had, essentially, the same agenda as the *Jahrbuch*, the latter aimed at maintaining the completeness of the coverage and the classification of all articles in each calendar year, whereas *Zentralblatt* put more emphasis on the promptness of the reviews and the international aspect.

“The initiative for the foundation of a new mathematical reviewing journal came from mathematicians Otto Neugebauer, Richard Courant, and Harald Bohr, together with the publisher Ferdinand Springer. The rapidly growing number of newly published mathematics works in the 1920s and the scientists need for obtaining quick information on recent material motivated the decision to create an alternative service to the *Jahrbuch*. [...]

collaborating with them.

His work with Busemann focused on Lebesgue integration [Feller 1934a] and on differential geometry resulting in a series of papers [*Feller 1935a, *Feller 1935b, *Feller 1936a, Feller 1936b]. Erhard Scholz discusses their contribution to the surface theory later in these *Selecta* [39].

Neugebauer recruited Feller to write for *Zentralblatt*, where we find eight of his contributions written in the years 1933-38. Two reviews, of Kolmogorov's seminal 1933 *Grundbegriffe* [Zbl. 0007.21601], and Khinchine's *Asymptotische Gesetze* [Zbl. 0007.21601] are particularly noteworthy: Both are enthusiastically friendly, in spite of the prevailing neutral tone of *Zentralblatt's* entries, and show Feller's familiarity with the new measure-theoretic probability theory. It should be emphasized that in 1933 Kolmogorov's work was just one more attempt to axiomatize probability theory. Feller was among the first authors who fully recognized the importance of this approach and adopted it throughout his career.

In 1934, with Harald Bohr serving as an intermediary, Feller obtained a position with Harald Cramér's group in Stockholm,

and stayed on in Stockholm for five years. He made a great number of Swedish friends, collaborating with economists and biologists as well as with the members of our [Cramér's] probabilistic group. He had studied in Göttingen and was well initiated in the great traditions of this mathematical center. We tried hard to get a permanent position for him in Sweden, but in those years before the war this was next to impossible, and it was with great regret that we saw him leave [in 1939] for the United States, where an outstanding career was awaiting him.³⁹

With his already published work in probability theory and analysis, and papers on differential geometry, Feller had choices to make, and the contact with Cramér tipped the scale in favour of probability theory. During his Stockholm years Feller wrote two seminal papers which would establish his prominent position as a probabilist: his work on limit theorems [Feller 1935c], where he found the necessary and sufficient conditions for the validity of the Central Limit Theorem (CLT), and his extension [Feller 1936c] of the results of Kolmogorov's *Analytische Methoden* paper [27], which provided the foundations of the theory of Markov processes. The mathematical methods used in both papers reflected his strong analysis and PDE background which he had acquired in Göttingen. There was more work on limit theorems [Feller 1937a] (influenced by Feller's interaction with Marcel Riesz), the paper on the weak law of large numbers [Feller 1937b] (dedicated to Harald Bohr and written during Feller's February 1937 visit to Lund), and another on infinite divisibility [Feller 1939d]; their mathematical contents is scrutinized in detail in Hans Fischer's essay later in these *Selecta* [16].

At the memorable Geneva *Colloque sur le Calcul des Probabilités*⁴⁰ Cramér and Feller met Neyman who "gave a lecture on his theory of confidence intervals, which was then something quite new", and met with a skeptical reception;⁴¹ Feller, however, immediately took up this topic in [*Feller 1938b].⁴² His own contribution to the colloquium [*Feller 1938a]⁴³ was a comparison of the von Mises', Tornier's and Kolmogorov's approaches to the foundations of probability theory; he continued this theme in [Feller 1939c]. Interestingly enough, after all the Tornier's political scheming in Kiel, Feller unflinchingly gave him the mathematical credit due. A critical assessment of this work can be found, again, in Fischer's essay [15]. At the same time, clearly with Kolmogorov

³⁹Neugebauer directed the new periodical for several years until the political situation in Germany made his position as editor-in-chief unsustainable. In 1933, shortly after the Nazi party rose to power, a law was enacted which banned Jews and political enemies from holding jobs as civil servants. A call to dismiss Courant, Neugebauer, Landau, Bernays, and Noether appeared in a local newspaper and, soon afterwards, Courant escaped to the UK and later moved to New York. Due to this pressure, Neugebauer decided to resign from his post at Göttingen University and in 1934 took up a professorship in Copenhagen, from where he continued his work for *Zentralblatt*.

⁴⁰The struggle to produce the reviewing journal became more difficult throughout this period, however, for the Nazis tried to influence the editorial policy. Neugebauer eventually gave up his position as editor-in-chief in 1938 after a series of incidents, including Levi-Civita being dismissed from the editorial board without his knowledge."

³⁹Cramér [8, p. 519]

⁴⁰11–16 October 1937, the proceedings are [18].

⁴¹Cramér [8, p. 528]

⁴²Also, see Neyman [30, p. 57, Footnote (3)].

⁴³Unfortunately, this important contribution cannot be reprinted in these *Selecta* because of the fees demanded by the publisher (Hermann, Paris).



Picture 4: Clara and William Feller on the train in the 1930s. Photograph courtesy of Joanne Elliott.

[27] and [Feller 1936c], as well as Risser [36], in mind he came up with a probabilistic interpretation of Volterra’s theory of biological populations using discrete-time and continuous-time stochastic processes [Feller 1939a] (see the commentary by Baake and Wakolbinger later in these *Selecta* [2]). During his Stockholm years Feller also started to collaborate with applied scientists (see [*Feller 1938c, *Feller 1940a]). In [Feller 1939b] he solved a problem “proposed [...] by Conny Palm, of the telephone-administration of Stockholm”, in connection with studies of telephone traffic; this started a new line of research for Feller and he would return to this topic time and again, see the commentary by Maller [28] in these *Selecta*.

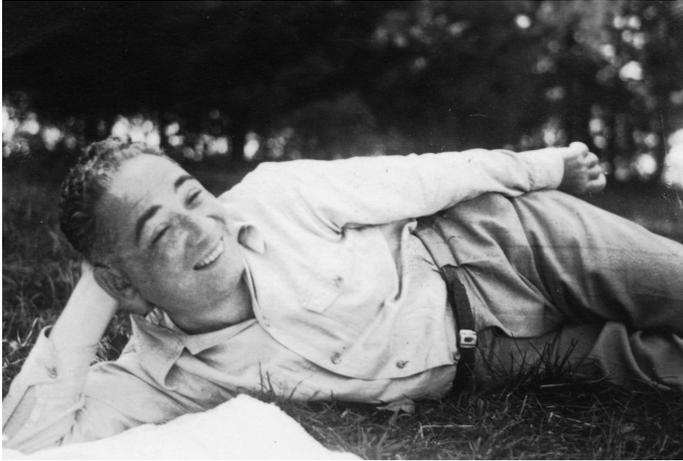
During Feller’s stay in Sweden, important events took place in his private life as well. On 27 July 1938 he married his former Kiel student, Clara Nielsen. Back in Zagreb, his parents passed away, his father in 1936 and his mother in 1938. Most of his siblings still resided in Zagreb, and we know that he remained in contact with his homeland. In particular, his two papers [*Feller 1934b, Feller 1939d] appeared first in Croatian in *Rad JAZU*, the journal of the Yugoslav (now: Croatian) Academy of Arts and Sciences, and a shorter German version (“Auszug”, meaning excerpt) with essentially the same mathematical content appeared in the international issue of *Rad*. At least since 1937, Feller had been a corresponding member of the Yugoslav Academy of Arts and Sciences.⁴⁴

The 1940s: Brown & Cornell

In the summer of 1939 the Fellers were on the move again. Since William could not get a permanent position in Sweden,⁴⁵ he accepted the offer of an associate professorship at Brown University in Providence, Rhode Island. At that time R. G. D. Richardson served as dean of the Graduate School at Brown, as well as Secretary of the American Mathematical Society (from 1921 to 1940). Richardson recognized early on the tremendous potential for growth of mathematics in the U.S. created by European emigré mathematicians and he was one of the “principal agents of the

⁴⁴On the titlepage of the *Rad*-version of [Feller 1939d] there is a note, “Napisao član dopisnik” [“written by corresponding member”], and the acceptance date (22 November 1937) is given. This, curiously, only appears in the original Croatian version of [Feller 1939d], cf. also Seneta [40, p. 87 *f.*] and Žubrinić [47, p. 45 *f.*; he confirms that Feller became corresponding member in 1937].

⁴⁵Possibly, also because of the more or less latent anti-Semitism in Sweden, see the comments and excerpts from Feller’s letters from 1934 and 1939 in Siegmund-Schultze [41, pp. 135 *f.*].



Picture 5: William relaxing on the grass during his Cornell days. Photograph courtesy of Joanne Elliott.

mathematicians in aiding emigrés”.⁴⁶ He mobilized various institutions in his efforts, including the well-known *Emergency Committee in Aid of Displaced German* [later: *European*] *Scholars*, and was instrumental in creating permanent positions at Brown University for W. Feller, O. Neugebauer, W. Prager and J.D. Tamarkin. These actions eventually led to the foundation of a distinguished applied mathematics division at Brown. Wartime Brown was an exciting place for mathematics: Run by Richardson, and with Tamarkin and then Prager as scientific directors, the programme focussing on fluid dynamics, elasticity, and PDEs was set up;

it included Feller, Prager, and Tamarkin, who had Brown appointments, and at one time or another, Stefan Bergman, K. O. Friedrichs, Witold Hurewicz, Charles Loewner, F. D. Murnaghan, I. S. Solonikoff, Richard von Mises, Stefan E. Warschawski, Antoni Zygmund and myself [Lipman Bers].⁴⁷

In 1941 Richardson launched the *Brown University Summer Session for Advanced Instruction and Research in Mechanics* (June 23 – September 13, 1941), and Tamarkin and Feller contributed with their lecture notes [*Feller 1941b] on PDEs. Ironically, Richardson’s move torpedoed initially a parallel Courant’s initiative to set up a center for basic and applied mathematics at New York University.⁴⁸ Mathematically, Feller’s work fit well into Richardson’s plans at Brown. There are several papers [*Feller 1940b, *Feller 1940d] where Feller engages with a non-mathematical (scientific) audience, and one of his notes [*Feller 1943a] appears to be specifically written for the new Brown University journal *Quarterly of Applied Mathematics*. By 1941 Feller’s first graduate student (co-mentored with Tamarkin) George E. Forsythe had defended his dissertation on summability methods. Later, Forsythe would become the founder of the Computer Science Department at Stanford. These developments were just a small component in a vast web of strong influences that Feller and other immigrant mathematicians have exerted on the American scientific environment after World War II; a thorough discussion of this historical phenomenon can be found in Siegmund-Schultze [41].

The main thrust of Feller’s research continued to involve limit theorems and Markov processes. In [Feller 1943c] he strengthens the results of Kolmogorov’s paper on the law of the iterated logarithm (LIL), giving necessary and sufficient criteria for norming functions to be in Lévy’s upper and lower classes. Feller would come back to this theme time and again. Mark Kac⁴⁹ called this paper a “veritable *tour de force*”, and it can be argued that this paper still is unsurpassed in its completeness and its methods. The paper is essentially self-contained, relying only on his own previous work [Feller 1943b] where asymptotic estimates for the tails of sums of (bounded) independent random variables were obtained. In a major survey paper [Feller 1945b] he reviews developments on the LIL, CLT, and its “little brother”, the weak law of large numbers (WLLN); he was also interested in applications of the CLT and the WLLN, e.g. in connection with the St.

⁴⁶Reingold [35, pp. 178, 184]

⁴⁷Bers [3, p. 240]

⁴⁸Reingold [35, p. 197], Reid [34, p. 228 f.]

⁴⁹M. Kac: William Feller, *in memoriam* – reprinted in these Selecta.

Petersburg paradox [Feller 1945d], and normal approximations of the binomial law [Feller 1945a]. His paper on conditional probability functions [Feller 1940c] which determine a not necessarily diffusive Markov process completed his seminal contribution in [Feller 1936c], and actually introduced the term “Markoff process”. Later, Doob⁵⁰ would say that

Feller completely transformed the subject of Markov processes. Going beyond his 1935 paper, he put the analysis into a modern framework, applying semigroup theory [in the 1950s] to the semigroups generated by these processes. He observed that the appropriate boundary conditions for the parabolic differential equations governing the transition probabilities correspond on the one hand to the specification of the domains of the infinitesimal generators of the semigroups and on the other hand to the conduct of the process trajectories at the boundaries of the process state spaces. In particular, he found a beautiful perspicuous canonical form for the infinitesimal generator of a one dimensional diffusion. In this work, he was a pioneer yet frequently obtained definitive results.

The geometry papers [Feller 1942], [Feller 1945c] (jointly with H. Busemann) were the last *contributions to geometry*; both continued his earlier work. One could argue that during his stay at Brown Feller mostly was adding finishing touches to his earlier achievements. The only paper which contained completely new material was his publication on renewal theory [Feller 1941a]. The reason could have been the new environment he found himself in, as well as various not research-related activities he was involved in during those years.

Around the year 1938 several mathematicians started a campaign for an independent American abstracting journal modelled on the *Zentralblatt* (Zbl) which was compromised by Nazi racist policies. By the end of May 1939 the name *Mathematical Reviews* (MR) was already in use and O. Neugebauer (who had been editing *Zentralblatt* from 1931 to 1934 from Göttingen, and then from 1934 till 1938 from Copenhagen) and J. D. Tamarkin were approached with the request to serve as founding editors; both were employed by Brown University and Brown was also the first location for the editorial office (until 1951). The first issue of MR saw the light in January 1940, covering articles published from July 1939 onwards.

Dr Willy [*sic!*] Feller [was appointed] to be technical assistant to the editors for a three year term effective 1 July 1939”. [...] The name of Feller first appears, with the title Executive Editor in vol. 5 (1944).⁵¹

In fact, Feller was Executive Editor from 1941 to 1945 and served in the editorial committee from 1948 until 1953.⁵² Since there was not much staff besides the editors, working for MR was quite time-consuming. Many reviews were actually written by the editors themselves, and a reviewer search on the MathSciNet database reveals that Feller wrote 843 abstracts in 14 years (1940–1954). On top of that he served as President of the Institute of Mathematical Statistics during 1947.

Towards the end of World War II, in 1944, William Feller became a US citizen. By 1945 he had quit as Executive Editor of MR, and soon thereafter decided to leave Brown University in 1945. He accepted a (full) professorship at Cornell University, where he stayed until 1950. During those five years at Cornell, Mark Kac and Richard Feynman were his colleagues, and K. L. Chung, G. A. Hunt and G. Elfving were among the many permanent and visiting staff members. Most of his articles at the time dealt with limit theorems [Feller 1946a, Feller 1946b], [Feller 1948a] (containing an interesting foray into statistics) and he developed important ideas regarding fluctuations and recurrent events [Feller 1949a, Feller 1949b, Feller 1949d]. In the paper [Feller 1949c] presented at the first Berkeley Symposium Feller collected and surveyed, for the first time, his research interests in the theory of Markov processes. Towards the end of his stay at Cornell, Feller had completed the first edition of the first volume of his landmark monograph, *An Introduction to Probability Theory and its Applications* [*Feller 1950]. Volume 1 is ‘elementary’ in the sense that it does not use measure theory. The more advanced parts of the theory would appear later in the second volume, but it is interesting to note that many of Feller’s recent research results were directly included. Work on this monograph must have started several years earlier. Indeed, in

⁵⁰J. L. Doob: William Feller and twentieth century probability. Reprinted in these Selecta.

⁵¹Pitcher [32, p. 72]

⁵²Pitcher [32]



Picture 6: Clara and William Feller around 1950. Photograph courtesy of Joanne Elliott.

a letter to Borge Jessen⁵³ Feller writes “I am writing myself a book which is to incorporate the mathematical theory of probability with many applications and we are just mimeographing the first few chapters.” From this period also comes the anecdote which was later told by J. Doob:

While writing my book [i.e. Doob: Stochastic Processes, 1953] I had an argument with Feller. He asserted that everyone said “random variable” and I asserted that everyone said “chance variable”. We obviously had to use the same name in our books, so we decided the issue by a stochastic procedure. That is, we tossed for it and he won.⁵⁴

The Princeton Period 1950–1970

In the summer of 1950 Feller was appointed Eugene Higgins Professor of Mathematics at Princeton University. Shortly thereafter Feller gave a 45-minute address [Feller 1952c] in the *Statistical Mechanics* section at the 1950 International Congress of Mathematicians, which took place between 30 August and 6 September 1950 in Cambridge, Massachusetts. Feller talked about recent trends in the theory of diffusion processes, covering some familiar terrain including his and Kolmogorov’s work on Markov processes, but now clad it in the new language of semigroup theory. Moreover, he explicitly mentioned Itô’s approach, Bochner’s subordination, and Riesz’ fractional derivatives. At that time this was truly visionary, mapping out a programme which would combine probability theory, (partial) differential equations and functional analysis.

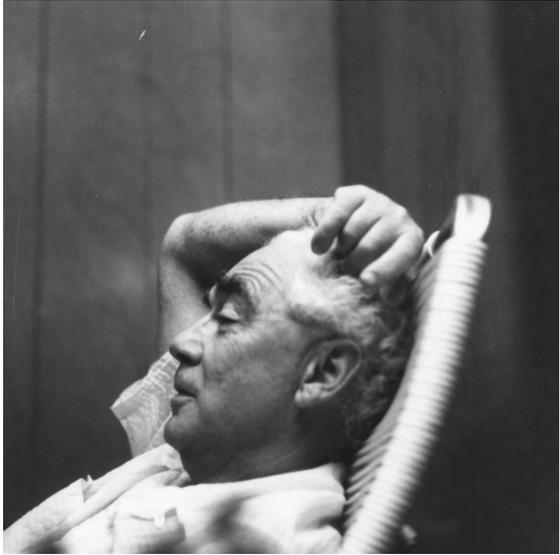
The Princeton years, in particular the 1950s, marked the second extremely innovative period in Feller’s research. He introduced semigroups into probability theory (following Bochner [6] and Yosida [46])⁵⁵ and started discussing processes on subsets of \mathbb{R} (or \mathbb{R}^d), leading to parabolic initial value problems with additional boundary conditions. At the same time he understood (but rarely worked himself on)⁵⁶ the pathwise interpretation of diffusions offered by Itô’s approach. The collaboration of one of his students, H. P. McKean, with K. Itô, culminating in the celebrated

⁵³Typewritten letter [14] of Feller to Borge Jessen dated 2 April 1947

⁵⁴Snell, interview with Doob [43, p. 307]. Interestingly, this did not prevent Feller to speak of “chance processes” in [*Feller 1961c].

⁵⁵Kendall writes in Reuter’s obituary: “Now Feller, in the preceding year [1951/52] had given a long course on aspects of the Hille–Yosida theory of semigroups in the context of probability theory” and he goes on how “semigroup analysis” helped him (Kendall) to understand and work (with Harry Reuter) on general Markov chains [25, p.178].

⁵⁶The interpretation of the boundary conditions in terms of the processes in [Feller 1954b] is one notable exception. In general, however “[h]e was one of the first generation who thought probabilistically [...], but when it came to writing down any of his results for publication, he would chicken out and recast the mathematics in purely analytic terms” (Rota [38, p. 227]). The same pattern can already be observed in Feller’s early contributions to probability theory, see Fischer [16].



Picture 7: William Feller at his home in Princeton. Photograph courtesy of Joanne Elliott.

monograph [23], was certainly no accident, see also the discussions in Fukushima’s and Peskir’s essays [19, 31] in these Selecta.

In a series of papers⁵⁷ he develops semigroup theory with probabilistic applications in mind which would usually be phrased in terms of the Kolmogorov backward and forward equations. Very important contributions are contained in the 1953 papers where he relaxes the continuity conditions (both in space and time) which are needed for the backward and forward Kolmogorov equations. Even more important is his discovery in [Feller 1952a] that the corresponding adjoint of the backward equation leads to an “essentially new boundary condition”⁵⁸ of second order. This line of research led to the general characterization of boundary conditions for one-dimensional diffusion operators which appeared in a cascade of papers⁵⁹; this body of work certainly is Feller’s most cited contribution to pure mathematics, see the essays by Fukushima [19], Jacob [24] and Peskir [31] in these Selecta.

At about the same time, building on his ideas from [Feller 1939a], he presents a groundbreaking paper at the Second Berkeley Symposium on Mathematical Statistics and Probability [Feller 1951d], paving the way for non-trivial applications of stochastic processes, in particular in genetics. At the end of the decade, he connects his diffusion and semigroup theory with birth and death processes [Feller 1959b] – an idea which was already present in the 1939 paper. Feller’s pioneering work on mathematical biology is among his most influential contributions to science, see the commentary by Baake and Wakolbinger [2] included in these Selecta.

At Princeton he was able to attract many exceptionally talented graduate students and young researchers to either work with him or to be strongly influenced by his work. Among his Princeton PhD students were Patrick Billingsley (1955), Henry McKean (1955), Hale Trotter (1956), Frank Knight (1956), David Freedman (1960), Lawrence Shepp (1961), Martin Silverstein (1965), Benjamin Weiss (1965) and Loren Pitt (1967); the Mathematics Genealogy Project⁶⁰ lists 22 students and 1043 descendants. Many established international visitors were attracted to Princeton as well. For example, Kiyosi Itô visited the Institute for Advanced Study in Princeton from 1954 to 1956, and almost immediately started working with Henry McKean, then one of Feller’s graduate students, and soon they both started exchanging their ideas with Feller. Their celebrated work on diffusion processes was in many ways influenced by Feller’s programme: In Itô and McKean’s words “W. Feller has our best thanks, his ideas run through the whole book.”⁶¹ In his MR book review, S. Watanabe wrote:

⁵⁷notably in [Feller 1952a, Feller 1952b, Feller 1952e, Feller 1953a, Feller 1953b, Feller 1959c]

⁵⁸Yosida, review MR0047886 (13,948a) of [Feller 1952a].

⁵⁹including [Feller 1952a, Feller 1954a, Feller 1954b, Feller 1955a, Feller 1955b, Feller 1957a, Feller 1957d, Feller 1958]

⁶⁰genealogy.math.ndsu.nodak.edu/id.php?id=33019 (accessed 5 August 2014).

⁶¹Itô and McKean [23, p. XI]

Around 1955, W. Feller's work on linear diffusion [MR0047886(13,948a) [Feller 1952a]; MR0068082(16,824g) [Feller 1955a]; and many others], which was primarily of analytic character, spurred some outstanding probabilists (including the present authors [Itô, McKean]) to re-establishing Feller's results by more probabilistic methods, solving some conjectures presented by Feller (in his paper or privately) and, finally, studying the structure of the sample paths of linear diffusion profoundly.⁶²

This nicely illustrates how Feller handled his interactions with his students. He never competed with them head-on but carefully led them onto independent, but still related to his own work, paths of research.

The 1950s were not only an exceptionally productive period for Feller but they also marked a breakthrough in the appreciation of his work in a wider mathematical and scientific community. At the age of only 45 he got appointed to a named chair at Princeton University, which was just about to enter top rankings of the mathematical world, taking over the role Göttingen had played before the war. In 1958 he was a plenary lecturer at the International Congress of Mathematicians (ICM) in Edinburgh and in the same year he was named a Fellow of the American Academy of Arts and Sciences. Two years later, William Feller became an elected member of the National Academy of Sciences (USA)⁶³, and among the many signs of international recognition was his appointment to serve on the *Fields Medal Committee* for the 1966 ICM in Moscow.

Feller continued working on his benchmark monograph. The second, slightly enlarged, but substantially changed, edition of Volume 1 appeared in 1957 and the preparations for the much more advanced Volume 2 (which would appear in 1966) must have been in full swing. The content of both volumes reflects Feller's research interests, for example, the material from the papers on fluctuations and coin tossing [Feller 1949b, *Feller 1951c, *Feller 1957b, *Feller 1959a] have been included in Volume 1, while Volume 2 would establish semigroup theory as a tool in probability and it is still one of the most complete discussions of limit theorems.

The 1958 ICM at Edinburgh showcased Feller at the height of his career. In the one-hour plenary talk [Feller 1960] he summed up his boundary theory, reviewed its connections with Martin boundaries, and explained the intimate link between probability and potential theory. But after the Congress had adjourned he left the field which he had initiated over the past decade, and turned his attention again to limit theorems and applications. As in Stockholm, a quarter-century earlier, he started interacting again with scientists and engineers. The expository paper [*Feller 1961c] was aimed at engineering students and he returned to research in mathematical population biology and genetics: [Feller 1966b, Feller 1967c, *Feller 1969a, *Feller 1969f]. Jointly with S. Orey he studied renewal theorems for random walks [Feller 1961a, Feller 1961b], and strong ratio theorems [Feller 1966a]. In the field of limit theorems he returned to the topics he investigated in the decade starting in 1935, studying in a series of papers fluctuations and oscillations of series of random variables [Feller 1966c, *Feller 1969e, Feller 1970], and limit theorems for large deviations and the LIL [*Feller 1968c, Feller 1968d, Feller 1969d, Feller 1969c]. In connection with limit theorems he contributed to Tauberian theory (for Laplace transforms) and the theory of regular variation [Feller 1963, Feller 1967a, *Feller 1968a] and [Feller 1969b]. A discussion of the mathematical contents of those papers can be found in Maller's commentary [28] in these *Selecta*.

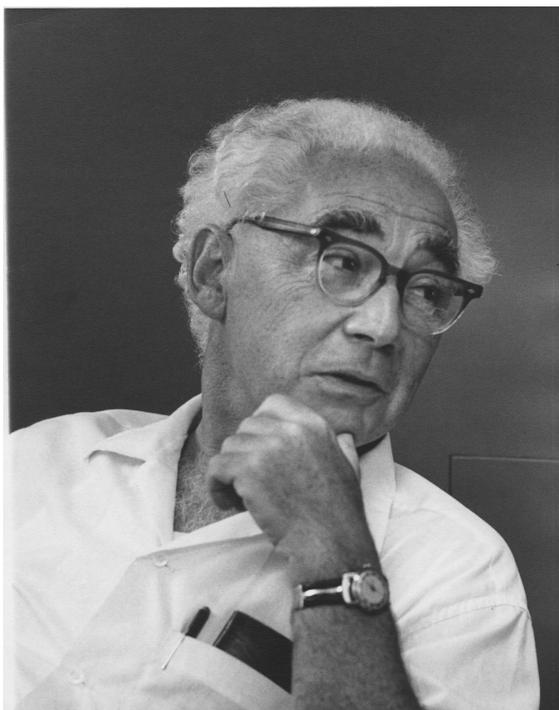
By the end of 1965 Feller had completed the first edition of the second volume of his exceptional monograph on probability and statistics. He would continue working on the next edition until the end of his life, and it is thanks to many of his former graduate students and collaborators that the last edition was eventually completed after his death. In 1966 Feller was also awarded the title of permanent visiting professor at the Rockefeller University in New York, NY. The university, having recently changed its name from Rockefeller Institute, was expanding its mission and started to hire faculty with expertise in physics and mathematics. Feller's collaborators there included T. Dobzhansky⁶⁴, among others, and the paper [Feller 1966b] was the first of Feller's publications with both affiliations, Rockefeller and Princeton University.⁶⁵ During that time Feller

⁶²S. Watanabe, review MR 0199891 (33 #8031) of the monograph [23] by Itô and McKean.

⁶³Rosenblatt [37, p. 12]

⁶⁴Theodosius Dobzhanski was one of the founding fathers of what is now known as *evolutionary synthesis* within modern evolutionary biology. It is worth pointing out that Feller's publications in mathematical biology were strongly influenced by his work.

⁶⁵Rosenblatt [37, p. 12]



Picture 8: One of the portrait pictures taken by R. W. Collier at IBM’s Thomas J. Watson Research Center, where Feller served as a consultant for many years. These photographs were also used by Monroe Donsker’s wife, Mary, to paint Feller’s watercolor portrait which is in the possession of Dr. Elliot. Photograph courtesy of Joanne Elliott.

also worked on the the third edition of Volume 1 of his probability textbook which appeared in 1968 [*Feller 1968e] (with a corrected reprint in 1970); again it differs considerably from the two earlier editions.

William Feller is remembered not only for his scientific achievements and his monographs, but lives on in the memories of many contemporaries as a “man of wide interests, with a profound knowledge of subjects like history and literature, and a great love of music.”⁶⁶

Feller was an ebullient man, who would rather be wrong than undecided, and who preferred getting a hearing for his views to getting applauded for them. [...] He spoke loudly, very fast, with a strong Yugoslav accent, with wit and charm and understanding.⁶⁷

He was short, compact, with a mop of wooly gray hair, irrepressible. In conversation quick, always ready with an opinion (or two) addicted to exaggeration. If you knew the code, you applied the “Feller factor” (discount by 90%) [...] [he was] so full of fun.⁶⁸

As a lecturer he was “loud and entertaining” and, although his proofs were not always correct or complete, the underlying idea was usually sound.⁶⁹ Coming to the ‘end’ of such an incomplete proof, Feller would stare out at the people in the class “with a set chin and an earnest look on his face, trying to elicit a slight nod from people, and he would often get such a nod.”⁷⁰ In this context Mark Kac coined the expression “proof by intimidation”.⁷¹ Feller did not adopt Landau’s Definition – Theorem – Proof style for his lectures and he did not routinely use the headlines “Theorem” or “Proof” on the blackboard.⁷²

We do not know when exactly Feller learned about his terminal illness, but we do know that he was aware of it for a while:

⁶⁶Cramér [7, p. 436]

⁶⁷Halmos [20, p. 94]

⁶⁸McKean quoted in Rosenblatt [37, pp. 13 f.]

⁶⁹Rota [38, p. 227]

⁷⁰Knapp [26]

⁷¹Rota [38, pp. 227 f.]. Knapp [26] confirms that the phrase “proof by intimidation” was actually used in Feller’s class in the academic year 1962/63, but he does not remember who had introduced it; it may well have been Feller’s own joke on various ways to prove things.

⁷²Knapp [26]



Picture 9: Clara Feller around 1960. Photograph courtesy of Joanne Elliott.

Having accepted the verdict himself he tried to make it easy for all of us to accept it too. He behaved so naturally and he took such interest in things around him that he made us almost forget from time to time that he was mortally ill.⁷³

William Feller died of cancer in a New York hospital on 14 January 1970. He did not live long enough to be present at the award ceremony for perhaps the most distinguished of his many recognitions, the *National Medal of Science* for 1969. His widow Clara received the award at the White House on 16 February 1970. After her husband's death, Clara continued her engagement with the mathematical community, and served as the Technical Editor for the *Annals of Mathematics* taking care of the Volumes 94/95 (1971/72). The second edition of Volume 2 of Feller's textbook [*Feller 1971] appeared posthumously in 1971; “[t]he manuscript had been finished at the time of the author's death but no proofs had been received” and proofreading, indexing and final touches were done by Feller's students.⁷⁴

Acknowledgement. Many colleagues supported us when writing this biographical sketch. In particular we would like to thank Christian Berg for supplying Feller's letters to Borge Jessen, Joanne Elliott for sharing with us her recollections and supplying many photographs of Feller, Hans Fischer, Niels Jacob, Joseph Rosenblatt and Ken-iti Sato for critical remarks and stimulating discussions. Tony Knapp kindly told us about Feller's teaching at Princeton and he shared with us his personal lecture notes of some of Feller's classes. Our Croatian colleagues, Nikola Sandrić, Hrvoje Šikić and Zoran Vondraček helped us with many documents and sources in Croatian which would otherwise have been inaccessible to us. Part of this exposition is based on Hrvoje Šikić's article [42] on Feller. We are also indebted to Ulrich Hunger from the *Staats- und Universitätsbibliothek Göttingen* who provided scans of the *Promotionsakte*, to Ms. Christiane Weber of TU Dresden for digitizing the photographs and to Ms. Sarah Hamdouchi of the *Einwohnermeldeamt* Flensburg, Germany, for checking the birth register of the city of Flensburg.

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⁷³Mark Kac in Rosenblatt [37, p. 13].

⁷⁴[*Feller 1971, p. xi]; J. Goldman, A. Grunbaum, H. McKean, L. Pitt and A. Pittenger are explicitly mentioned.

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Prof. René L. Schilling
Fachrichtung Mathematik
Institut für Mathematische Stochastik
Technische Universität Dresden
Helmholtzstraße 10
01069 Dresden
Germany
rene.schilling@tu-dresden.de

Prof. Wojbor A. Woyczyński
Department of Mathematics,
Applied Mathematics and Statistics
Case Western Reserve University
10900 Euclid Avenue
Cleveland, Ohio 44106
U.S.A.
waw@case.edu