

**Michel Talagrand: *Upper and Lower Bounds for Stochastic Processes. Modern Methods and Classical Problems.***

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Bounds for Gaussian processes is one of the recurrent themes in the oeuvre of Michel Talagrand. The monograph under review—which may well be his *opus magnum* on bounds for stochastic processes—is entirely devoted to the study of the supremum of a stochastic process  $(X_t)_{t \in T}$  indexed by a general index set  $(T, d)$  (usually seen as a finite or infinite metric space). The aim is to present a unified approach to the problem of boundedness (and the then (!) *simple matter to study continuity* [Chapter 1, p. 12]) for Gaussian and many other classes of mostly non-homogenous stochastic processes, e.g. Bernoulli processes, random Fourier series,  $p$ -stable processes and infinitely divisible processes. The principal tool, Kolmogorov’s chaining idea which has been refined and used by Dudley for his maximal ‘entropic’ estimate for Gaussian processes, has been earlier exposed by Talagrand in *The Generic Chaining* [Springer, Berlin 2005], but the present approach is much more general and far-reaching, it includes most of the material of the predecessor book and takes into account a wealth of material from Talagrand’s other classic (joint with M. Ledoux) *Probability in Banach Spaces* [Springer, Berlin, 1991]. The reader is assumed to be familiar with classical methods from the theory of processes (e.g. Kolmogorov–Chentsov criteria, the Garsia–Rodemich–Rumsey lemma or martingale methods), but as far as new ideas and developments since the 1970s are concerned (many of them were pioneered by Talagrand), the present volume is self-contained.

Rather than trying to do the impossible in this brief review (i.e. to do justice to this work, giving a true picture of the content or at least an overview of the material covered in the 600-odd pages) let me point out that this is a most scholarly account written by a master of the field. Each chapter begins with an explanation of the overall philosophy and gives a brief survey of the things to come. This makes a rewarding read for everyone with a sound probability background, for the specialist it is even entertaining and most inspiring. Apart from the chapters devoted to special classes of processes (Chapters 3–11, 14), the first two introductory chapters (pp. 1–74) are a wonderful introduction into the topics and methods of this book, while Chapters 12 & 13 and 15 & 16 provide (sometimes unexpected) applications of the bounds and inequalities for stochastic processes. There are many areas where Talagrand has a very personal view of things which, I am sure, will be food for thought for future generations of probabilists.

René L. Schilling  
Institut für Stochastik

TU Dresden  
D-01062 Dresden, Germany  
[rene.schilling@tu-dresden.de](mailto:rene.schilling@tu-dresden.de)