Stein, Elias M.; Shakarchi, Rami: *Real Analysis. Measure Theory, Integration and Hilbert Spaces*. Princeton Lectures in Analysis Vol. 3. Princeton University Press, Princeton, Oxford 2005, xix + 402 pp., £38.95, ISBN 0-691-11386-6.

The Princeton Lectures in Analysis are a series of four one-semester courses taught at Princeton University. The lectures cover Fourier Analysis, Complex Analysis, Real Analysis and Functional Analysis. Their principal aim is to introduce the core material of analysis at graduate level in an integrated manner, emphasizing interrelations between the fields (both from a modern and historical perspective) and the wide applicability of analysis in other fields of mathematics and the sciences.

The book on *Real Analysis* is the third instalment of the series; it is a rather classical text, introducing gently into the theory of (abstract) Lebesgue measure and integration—the first two volumes of the series on Fourier and Complex Analysis were based on Riemann's integral. At the beginning there is a brief historical motivation, reminding the reader on some of the problems that stood at the origin of Lebesgue's theory: the question which functions can be represented by Fourier series, the problem how to measure the length of a curve and the relation between integrals and differentials.

The first 3 chapters of the book treat Lebesgue measure, Lebesgue integration and Lebesgue's differentiation theorem. The approach is measure-theoretic, i.e. first one introduces Lebesgue measure on rectangles, then it is extended via the Carathéodory construction onto the Borel sets. The integral is first defined for step-functions and then, using monotone convergence, for positive measurable and finally integrable functions with arbitrary sign. Highlights are the treatment of the Brunn-Minkowski inequality at the end of Chapter 1 and the whole of Chapter 3, where the interplay of differentiation and integration is examined. Chapter 4 is on Hilbert spaces. Motivated by the space L^2 the authors introduce the standard machinery for separable Hilbert spaces and give a treatment which is geared towards orthonormal systems, in particular, Fourier series. Things get really interesting in the fifth chapter where examples of Hilbert space methods are given, e.g. Hardy spaces on the upper complex half-plane (thus referring to volume 2 of the series), a Fourier-analytic treatment of partial differential operators with constant coefficients and the Dirichlet principle.

Having initiated the reader to the classical Lebesgue theory, the abstract measure and integration theory for arbitrary Borel measures on $\mathbb{R}^n$ can be treated rather quickly in Chapter 6. Again the presentation uses Carathéodory's extension theorem and the passage from measure to integral is via step functions. Product measures, Fubini's theorem and Lebesgue-Stieltjes integrals are now just examples of the general theory. The proof of the important but always cumbersome Lebesgue-Radon-Nikodým theorem follows von Neumann and is very clear and sleek. As a bonus the authors cover the mean, maximal and pointwise ergodic theorems and the spectral theorem for bounded operators on a Hilbert space. The last chapter of the book is on Hausdorff measure, Hausdorff dimension and fractals. The presentation only scratches this subject at the surface, the most interesting topic being the construction of a Besicovitch set.

Overall this book is a nicely written, logical continuation of the series. The topics and their presentation are closely linked to the first two volumes. In principle, the text should be accessible to everybody with a sound knowledge of analysis of one real variable. Because of its wealth of exercises and problems it will make a very stimulating supplementary reading for every course on measure and integration theory.

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