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Rindler, Filip: ***Calculus of Variations***. Universitext, Springer, Cham 2018, xii + 444 pp., US-$ 79.99, £39.99, EUR 71.68, ISBN 978-3-319-77636-1.

Variational principles appear in many real-life situations and many mathematical models become simpler and more elegant by introducing some kind of variational principle. Roughly speaking this means to optimize some appropriate 'energy' functional. The book under review focusses on minimization problems for integral functionals of the form

$$\mathcal{F}[u] = \int_\Omega f(x, u(x), \nabla u(x))\,dx$$

where $\Omega \subset \mathbb{R}^d$ is a bounded domain and $u : \Omega \to \mathbb{R}^n$ some function from a suitable function space, e.g. a Sobolev space. Classical examples are Bernoulli's Brachystrochrone Problem or the Isoperimetric Problem, but variational methods can also be used in Electrostatics, for Optimal Saving and Consumption, Sailing against the Wind, or to describe Microstructure in Crystals, Composite Materials and Phase Transitions. Already this list of examples indicates that the calculus of variations is both a theory as well as a collection of methods to treat concrete problems arising in the theory of (quite often nonlinear) partial differential equations.

The present book has two parts, a basic course (Chapters 1–7, pages 1–182) which may well be a MSc-level lecture or reading course and a selection of advanced material (Chapters 8–13) which can be read selectively, giving access to research-level topics. The required prerequisites are elements of (vector) analysis, measure theory, functional analysis and Sobolev spaces. These preliminaries are recalled in a short 18-page appendix, without proofs but with precise references to the literature. The focus of the text is on direct methods to find the minimizer(s), i.e. the classical detour via necessary conditions and Euler's equations is avoided and only treated in Chapter 3 on *Variations*. It is a welcome feature that the author uses modern methods, whenever possible and practicable; this means that the text can start from scratch and comes relatively quickly, in the second half, to the forefront of modern research. I have never seen a text which successfully treats Young measures, relaxation and the rather modern concept of polyconvexity at this level. Of course, one has to pay a price for this: occasionally some deeper theorem is quoted without proof and not all statements appear in their utmost generality. From my point of view this is not a disadvantage: after all, this is a textbook and not a research monograph. Part 2 contains a selection of topics from the research literature of the past 30 years. There are chapters on Rigidity (mainly on differential inclusions and compensated compactness), Microstructure, Singularities (introducing measure-valued and BV solutions), Generalized Young Measures and Γ-convergence (with a focus on homogenization). Each chapter starts with an introductory section where the problem(s) are illustrated using simple settings, and at the end we find a short section on the history of the material and further developments, as well as a problem section.

Summing up, this is a well-written and most welcome addition to the (textbook) literature.

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