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V. Mandrekar, B. Rüdiger: *Stochastic Integration in Banach Spaces. Theory and Applications.* Springer, Probability Theory and Stochastic Modelling **73**, Cham 2015, viii + 211 pp., €74.89, SFR 93,50, £62,99, US-$ 89,99 (RRP). ISBN 978-3-319-12852-8.

Over the past decade stochastic partial differential equations (SPDE) driven by continuous and discontinuous noises have attracted increasing attention. One approach is to understand an SPDE as an infinite-dimensional SDE, driven by an infinite-dimensional (often Hilbert or Banach space-valued) stochastic process. This requires the notion of a stochastic (Itô) integral driven by an infinite-dimensional noise and the corresponding stochastic calculus. Despite the wealth of research publications in the field, there are only few monographs on infinite dimensional stochastic integration driven by continuous processes and even less for integrals with non-continuous drivers—actually the books by Metivier & Pellaumail [*Stochastic integration* (1980) Zbl 0463.60004] and Métivier [*Semimartingales: A course on stochastic processes* (1988) Zbl 0503.60054] seem to be the only sources for the general case. Usually, authors rely on *ad hoc* constructions or refer to various journal publications.

The present monograph is an attempt to fill this gap, focussing on Banach-space valued stochastic integrals driven by jump-type infinite-dimensional stochastic processes, in particular, by infinite-dimensional Lévy processes. Inspired by Rosiński's 1984 paper [Random integrals of Banach space valued functions. *Stud. Math.* **78** 15–38 Zbl 0559.60050] the authors use an approach based on (compensated) Poisson point processes, extending the presentations of Skorokhod [*Studies in the theory of random processes* (1965) Zbl 0146.37701] and Ikeda & Watabanbe [*Stochastic differential equations and diffusion processes* 2nd ed (1989) Zbl 0684.60040] to a Banach space setting. Most importantly, it is shown that this approach and Metivier's notion of stochastic integration are essentially compatible.

The presentation is largely self-contained, but a certain degree of sophistication on the side of the readers is assumed, e.g. the notion of Bochner and Pettis integrals or measurability concepts from the general theory of stochastic processes are only very briefly discussed, and some familiarity of probability in Banach spaces is certainly helpful. Having said this, the book comes rather quickly to the heart of the matter: the definition of Wiener (with deterministic integrands) and Itô (with predictable random integrands) integrals in Banach space based on compensated Poisson point processes. Using the Wiener integral, the authors are able to define Banach-space valued Lévy processes and more general martingales and their Lévy-Itô (or semimartingale) decompositions. This is then used to show that the martingale-based approach by Métivier is compatible with the present theory. A rather general Itô's formula is developed which is useful, in particular, in connection with Lévy-driven SPDEs. Having in mind applications, the authors discuss existence, uniqueness, (non-)Markovianity and dependence on the initial data of Banach-space valued SDEs and—as an application—SPDEs in Hilbert space. The latter are treated in the functional analytic way as in da Prato & Zabczyk [*Stochastic equations in infinite dimensions* 2nd ed (2014) Zbl 06315262] or Peszat & Zabzcyk [*Stochastic partial differential equations with Lévy noise. An evolution equation approach* (2007) Zbl 1205.60122] which requires a short excursion into semigroup theory. The last part of the monograph covers some applications of infinite-dimensional jump-type SDEs to mathematical finance, non-linear filtering and the stability of semilinear stochastic equations.

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