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Lowen, Steven Bradley; Teich, Malvin Carl: Fractal-based point processes. Wiley-Interscience, Wiley Series in Probability and Statistics, Hoboken NJ 2005, xxiv + 594 pp., US-$ 99.95, ISBN 0-471-38376-7.

The aim of this book is to give an integrated introduction to and exposition of fractals, stochastic point processes and their interplay; the presentation is geared towards graduate students and researchers in mathematics and the sciences. A *strong mathematical background and a solid grasp of probability theory* is required from the readers; the meaning of this is somewhat unclear as the text itself—the authors are electrical engineers by training and not mathematicians—is to some degree non-mathematical and quite often a juxtaposition of facts, formulae, recipes and graphics. In order to guarantee a *smooth flow of material*, all *lengthy* derivations *are relegated to* an appendix comprising some 50 pages. I could not find the keywords 'Definition' or 'Theorem' as one would expect from a rigorous mathematical text.

The book comprises 13 chapters, 3 appendices and a massive bibliography (mainly to the science literature). The appendices contain (A) some mathematical derivations, (B) solutions to all exercises in the text and (C) an index of notation. The text proper starts (chapter 1) with a short introduction to fractals, random fractals and stochastic processes. Chapter 2 gives details on fractals and their characteristics, with an emphasis on scaling and power-law behaviour, and lists the typical examples and non-examples of fractal behaviour. Somewhat of a mathematical curiosity, it is the capacity (or box-counting) dimension which is considered first and not Hausdorff dimension. Important for scientists, the origins of fractal behaviour in real life are also briefly discussed. In chapter 3 we encounter point processes (on the real line) and their characterisation. It is here where some elements of probability theory are explained to describe various probability laws (their expectation, variance, curtosis, (auto-)correlation, ...) associated with point processes. For later use, some techniques for the statistical analysis, e.g. the spectrum and wavelet variance methods, are introduced here. Chapters 4–10 are devoted to the description of the distributions and other properties of various classes of point processes (pp): Poisson pp (ordinary, doubly stochastic, compound), renewal pp, Lévy dust, fractal(-rate) point processes and various classes based on fractal pp such as fractional Brownian motion, other fractional Gaussian processes, fractal renewal processes or fractal shot noise. Usually the discussion involves only distributional properties, which is understandable as the book focusses on a description/simulation point of view. In chapter 11 basic *operations* with point processes are discussed, e.g. time-changes, (random) thinning-out, superposition to mention but a few. The last two chapters of the monograph are devoted to estimation of fractal point processes (chapter 12) and a concrete internet traffic model (chapter 13). The upshot of chapter 12 is that the authors favour the rate spectrum and the normalized (Haar-)wavelet variance methods to detect and estimate fractal behaviour; other methods are discussed as well. The last chapter concentrates on the question *how to model* internet traffic and gives a thorough discussion of various competing models.

Despite of all this criticism the book is still a valuable resource and a quarry for many a concrete result from real life.

René L. Schilling
FB 12 - Mathematik, Philipps-Universität Marburg, D-35032 Marburg, Germany
schilling@mathematik.uni-marburg.de