Andreas E. Kyprianou: *Fluctuations of Lévy Processes with Applications. Introductory Lectures.* Springer, Universitext, Heidelberg 2014. xviii + 455 pp., € 64.19, US-$ 79.99, £39.99. ISBN 978-3-642-37631-3.

This is the second edition of the author's popular textbook *Introductory Lectures on Fluctuations of Lévy Processes with Applications* [Springer, Berlin 2006; MR2250061 (2008a:60003)]. Compared to the first printing, the new edition contains about 25% new or substantially rewritten material. The most obvious changes are the newly added sections on *Spcecial and Complete Subordinators* [Section 5.6, pp. 138–145] and *Vigon's Theory of Philanthropy and More Examples* [Section 6.6, pp. 188–191] and the new Chapters 9 & 10 entitled *More on Scale Functions* and *Ruin Problems and Gerber–Shiu Theory*; on the other hand, the brief sections on generalised Ornstein–Uhlenbeck processes [1st ed: Section 8.6, pp. 231–233] and on stochastic games [1st ed: Section 9.6, pp. 260–268] are now gone, along with the rather complete solutions to the exercises (more than 60 pages in the first ed) which *has been replaced by a more terse set of hints*, comprising some 20-odd pages, *in response to the remarks of several colleagues* [p. x]. Broadly, the exercises following each chapter have not been changed too much, they range from relatively straightforward problems to guided prove-it-yourself theorems (often with original sources). Unfortunately, the *Epilogue* where new developments are summarized was not really updated and still shows the 2006 state of the art of the otherwise rapidly developing subject. Apart from these changes, there are only minor corrections and additions; a *caveat* for those readers who are familiar with the first edition is in order: Owing to Springer's new monograph design, some additional material and the occasional newly numbered equation, the original pagination and numbering have completely changed.

Despite these alterations, the character, scope and audience of the book are still the same. The book grew out of lectures pitched at an advanced undergraduate or beginning graduate audience, the prerequisite being a course on abstract Lebesgue integration and a good foundation in probability theory (including continuous-time martingales and basics of Brownian motion). The overall theme of this book is fluctuation theory of Lévy processes, and the principal applications—renewal theory, queueing theory, storage and risk theory—are chosen because they are related to fluctuation theory in the one or the other way. There is a substantial overlap both in the choice of topics and the mathematical approach with Bertoin's modern classic *Lévy Processes* [Cambridge University Press, Cambridge 1996; MR1406564 (98e:60117)], but Kyprianou's presentation is more geared towards students and much easier accessible for the novice. On the other hand, the style is informal, even colloquial—apt for lecture notes, more problematic for a monograph; for example, some fundamental concepts and definitions are introduced *en passant*, cf. the rather short discussion of infinite divisibility. At some places, a few (more) words on the historical background or alternative approaches (e.g. the construction of Lévy processes, the Markov nature of Lévy processes, potential theory) would have been helpful.

Let me briefly describe the content. The first three chapters (pp. 1–90) give a compact introduction to Lévy processes. Early on, the readers encounter the principal ex-

amples of Lévy processes (Poisson and compound Poisson processes, Brownian motion, Gamma and inverse Gaussian processes, stable Lévy processes) along with their applications or reincarnations (as Cramér-Lundberg risk processes, queues or continuous-state branching processes). The construction of general one-dimensional pure jump Lévy processes is through Poisson point processes—these are very briefly introduced—and the Lévy–Itô decomposition. From there onwards the presentation is closely focussed on fluctuation theory: Only those features of Lévy processes which will be needed later on are discussed, mainly, the variation of the paths, the Markov property, duality & time reversal, and exponential moments. Chapters 4–8 (pp. 91–255) form the core of the monograph. This is a very carefully written introduction to fluctuation theory and its probabilistic tools. As a warm-up for the things to come the author discusses storage theory (Chapter 4) for Lévy processes with bounded variation. In preparation of spectrally one-sided (i.e. having only positive or negative jumps) Lévy processes, the behaviour of subordinators at threshholds (jumping, creeping, over- & undershoots) is discussed in Chapter 5; distributional aspects are not touched upon. Extending the previous two chapters, the first passage behaviour of Lévy processes, including the author's own quintuple law, is described in Chapters 7 (for general Lévy proceses) and 8 (for spectrally one-sided Lévy processes). Here we find many original contributions by the author with various co-authors. Chapter 9, which is new in the second edition, contains more concrete examples of Wiener–Hopf factorisations and scale functions. The final four chapters are some kind of *denouement* containing various applications to risk theory, optimal stopping, branching processes and self-similar Markov processes, respectively. These chapters can be read independently, each containing different applications of the previously developed fluctuation identities. Already in the first edition, featuring only two concluding chapters, optimal stopping problems and continuous-state branching processes, these chapters did read like an addition rather than an integral part of the book, a *denouement* where all the tension built up in the previous parts is gone. Adding yet two more such chapters makes the final 150 pages of the second edition appear more like a variety bag—in stark contrast to the well-designed first two-thirds of the monograph.

Fluctuations of Lévy Processes is an interesting book and it is currently the best introduction (for the novice) to this important topic. The second edition features a few new additions, but the most important feature is that it brings the book back into print. On the other hand, this is one of the books where you should also keep the first edition (if you own it) since the second edition does not any longer have full solutions to the exercises.

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