Review commissioned by: Math Reviews

Khrennikov, Andrei: *Probability and Randomness: Quantum versus Classical.* Imperial College Press, London 2016. xvi + 282 pp., £ 91.07, US-$ 118.00 (RRP). ISBN 978-1-7832-6796-5.

This book grew out of a couple of lectures on the foundations of probability and randomness for physics students at Vienna's Institute for Quantum Optics and Quantum Information. The aim of the author is to provide a concise introduction to classical & quantum probability and randomness which is needed for a correct understanding of experimental results and measurements in quantum physics. He "*hope*[s] *that the book will serve as a textbook*" (p. x) on these topics as well as an introduction to interpretations of quantum mechanics.

It would be an unconventional textbook, indeed. The readers are required to have a modicum of knowledge of quantum theory and they learn some elementary facts of axiomatic (in the sense of Kolmogorov) probability theory, along with some Lebesgue measure theory. Moreover, the author revives the foundational debate of the 1930s ('von Mises vs. Kolmogorov' and 'frequentist probability vs. subjective probability' and tries to explain the difference of probability and randomness (Chapters 1–3). The short introduction to quantum formalism (Chapter 4) is difficult to access without prior knowledge of quantum mechanics, but it is needed in order to understand the heart of the matter which is exposed in Chapter 5: the relation between quantum probability and contextual probability. The latter means that "*all probabilities depend on complexes of physical conditions, contexts:* $P(E) = P_C(E)$*, where* C *is* [sic!] *experimental context. It is meaningless to speak about probability without determining a complex of physical conditions, context*" (p. 256). The author formalizes Feynman's interpretation of the probability structure of the classical two-slit experiment: since this is a multi-contextual structure, classical probability theory seems to be incapable of dealing with it properly. If one understands, however, contextual probability as (classical) conditional probability, the author shows how one can embed the outcomes of the two-slit experiment into a single probability space. This is achieved in Chapter 8 using, not surprisingly, a standard 'product of measure spaces' argument where the (unconditional) probability measure is explicitly constructed; unfortunately, the author fails to tell the reader what he is really doing. With this approach he deviates from – not to say: disproves – his earlier point of view aired in [Interpretations of Probability, 2nd ed, de Gruyter 2009; MR2494039] that this contextual probability is a "*non-Kolmogorovian probability*". Chapter 6 is "*most difficult for reading. It is about interpretations of quantum mechanics*" (p. ix). In a nutshell, the author tries to classify various interpretations of quantum mechanics: classical, information, statistical Copenhagen, quantum Bayesian (Khrennikov uses the witty

abbreviation QBism) or the Växjö interpretation. The conclusion is that his own Växjö interpretation unifies some of these theories and "*has a lot in common with QBism*" (p. x). The concluding chapter (Chapter 9) contains a very personal view on potential applications of quantum theory to the life sciences and real world problems; it also treats philosophical questions of the type: "*Have a cell, DNA or protein molecular a kind of free will?*" [sic!]. The text concludes with a presentation of the 2002 Växjö interpretation of quantum mechanics which is essentially a reprint of Sections 2–4 of the author's paper [Växjö interpretation of quantum mechanics. In: *Quantum Theory: Reconsideration of Foundations*. Ser. Math. Modelling, Växjö University Press, vol. 2, pp. 163–170.]

As already mentioned, specialist knowledge is needed in order to savour this monograph. Given the fact that it is interspersed with personal stories and philosophical considerations, treating mathematics sometimes gentleman-like, it is rather a popular science book for the erudite than a bona fide textbook. The author mentions rightly that the text is in places not an easy read, but his writing style doesn't make it easier for the reader. There are countless capital-letter abbreviations (QM, QP are easily guessed, but FTP 'formula of total probability', PCSFT 'prequantum classical statistical field theory' are more demanding, and there are more: SOC-POVM, QLRA, CHSH...) which are usually not listed in the index; occasionally, the explanation in the running text is *after* their first appearance. The narrative is disturbing, as it oscillates between a first-person singular ('I'), plural ('we') and the neutral ('one') perspective – often within a few lines. A more orthodox usage of the English language (along with a more careful proof-reading – the repeated misspelling in the bibliography of one of the book's heroes as "Kolmolgorov" is tale-telling) would have been helpful, especially when it comes to passages of a more philosophical nature where full command of the language is indeed needed to avoid ambiguities.

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