

Erschienen in: *Metrika*

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**Fristedt, B. and L. Gray: A Modern Approach to Probability Theory.** Birkhäuser Verlag (Probability and Its Applications) Boston, Basel, Berlin 1997. xx + 756 pages, DM 118,-. ISBN: 0-8176-3807-5

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This textbook by Fristedt and Gray is a volume in the Birkhäuser Series on *Probability and Its Applications*. It is intended to serve as introductory text for students of mathematics and related subjects. The prerequisites are deliberately kept at a low level, some naive probability, basic calculus, and a bit linear algebra will do. All other mathematical concepts and notions needed are either developed in the text (measure theory) or in an appendix (metric spaces, topology, etc.)

The text is divided into six Parts and 33 Chapters plus an Appendix. From a mathematical point of view, the division into a standard introduction into probability, roughly the first 500 pages, and selected topics on the following 200 pages, seems to be more natural; in fact, the introductory parts are much better suited for linear reading while the last few chapters invite to selective reading. This reveals the underlying concept: in the first section the foundations of the theory are laid, the second is thought as a brief introduction into more specialized fields, bridging the gap between textbook and research level.

Section one (pp. 1–488) contains the material usually covered in an introductory course of measure-theoretic probability theory at an advanced undergraduate or first-year graduate level. Starting with abstract (but not topological) measure and integration theory, probabilistic concepts are introduced: random variables, independence, sequences of iid random variables, basic Fourier analysis, classical limit theorems, various convergence concepts in probability, invariance principles & (real) Brownian motion, infinite divisibility, conditional probability, and (discrete-time) martingales. In the presentation of the material, random variables are consequently preferred to distributions. Worth being noticed is also the use of Etemadi's Lemma for an elegant proof of the strong law of large numbers (as well as at other places, e.g. for Lévy processes)—this is a rare event in the existing textbook literature—and the very clear and general treatment of triangular arrays and (1-dim.) infinite divisible distributions as limiting distributions. The presentation of different versions of the optional sampling theorem makes good reading, too.

Throughout this first section the reader is invited to study random walks in various settings. This is a nice preparation for the second part where several topics from discrete-time (Part 5, pp. 489–580) and continuous-time (Part 6, pp. 581–683) stochastic processes are treated. Part 5 contains a discussion of discrete-time renewal, Markovian, and exchangeable processes. Each of these topics is dealt with on some 20 pages. In Part 6 the reader is introduced to (Poisson) point processes, Lévy processes, (strong) Markov & Feller processes and the martingale problem, interacting particle systems, and, finally, diffusions and stochastic differential equations. Again, each theme gets roughly 20 pages space.

The breadth of topics included in this book is breathtaking and—in spite of the 750-odd pages the volume is containing—one is led to doubt either the profoundness or the honesty of the presentation. A closer look is necessary. *There are more than 1200 problems* [cf. p. xvii], and, as a matter of fact, many of them are used to ask the reader to complete essential steps in the proof of a major theorem—or even to furnish the proof by himself. Among the theorems thus proved are certain basic theorems of measure theory (existence & uniqueness of measures, convergence theorems [only Beppo Levi and Fatou are completely proved], chain rule for measures, Fubini’s theorem), but also the Truncation Lemma for iid sequences, the Glivenko-Cantelli Lemma, Strassen’s Law of the Iterated Logarithm for Brownian Motion, the Hewitt-Savage 0-1 law, Doob’s decomposition, Ergodicity of Markov/exchangeable sequences, Itô’s Formula for Brownian Motion, just to mention a few with a name attached. This shouldn’t happen in an introductory text. *[Some sort of ‘answers’ are available at the Web site* [p. xvii]; indeed, on the Birkhäuser World-Wide-Web home page one can download solutions, hints, etc. for some 300 of those problems. The idea, itself, is not bad. One should, however, think of situations where suitable computer or software equipment is simply not available. (By the way: is there a guarantee that Birkhäuser maintains this internet service for a reasonable time? For libraries, in particular, *reasonable* must not be confused with *as long as the book’s on sale.*)

This criticism may be, at least for the last 200 pages, somewhat harsh; the idea to bridge the gap between textbook and monograph should justify the one or the other omission. The problem, however, is that in the limited space devoted to so many topics none of them can be treated in a sufficient way. This is particularly annoying in the continuous-time theory: there is usually not much more material covered than the bare-hands construction of the processes or a rather elementary introduction to stochastic differential equations driven by a Wiener process. A common feature is also the *en passant* treatment of dimensions other than  $n = 1$  (already in the first

section of the book). No general construction of Markov processes is given (accomplished, however, for discrete time), martingale theory is transferred to continuous time just by defining the notion—with the remark that it is *quite easy to adapt the main results [...] to the continuous-time setting* [p. 630]; no statements, no proofs. We couldn't either find an explicit warning that *naive stochastic integration is impossible*. This would have been all the more important, as sometimes (correct) path-by-path Stieltjes interpretations of stochastic integrals w.r.t. a Brownian motion were employed. This can be very misleading for the inexperienced reader.

Although the last chapters are intended as some invitation to research they remain too shallow to accomplish this goal. It would have been a good idea either to include cross-references (in particular for further reading) in the text rather than only in a bibliography (ordered by the chapter) in the Appendix, or to comment on the reference texts in the bibliography.

From a technical point of view, the book is very carefully made (good paper, proper binding, nice typesetting, useful index) and the proofreading must certainly have been meticulous (we spotted only a handful misprints on 756 pages). The book itself, however, remains a *torso*, both in its intention—too many open problems for a rigorous introductory text—, and in its scope—a rather meagre *amuse gueule* of research topics.

Erlangen

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