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Chung, Kai Lai; Walsh, John B.: Markov Processes, Brownian Motion, and Time Symmetry. Second Edition. Springer, Grundlehren der mathematischen Wissenschaften Bd. **249**, New York 2005, xii + 431 pp., US-$ 129.00, EUR 109.95, ISBN 0-387-22026-7.

This monograph is a considerably extended second edition of Kai Lai Chung's classic *Lectures from Markov Processes to Brownian Motion* which appeared 1982 as volume 249 in the Springer Grundlehren Series, see MR 84c60091 or Zbl 0503.60073 for a review.

The new edition, now in joint authorship of K.L. Chung with J.B. Walsh, comprises two parts: part I (Chapters 1-5, pp. 1–232) is essentially a reprint of Chung's old book with minor corrections; part II (Chapters 6-15, pp. 233–420) is completely new and due to Walsh.

Since the first chapters are identical with the 1982 edition, we refer to the above mentioned excellent reviews in MR and ZMath. The only thing I want to mention on this part of the new edtition is that the publisher did a less than excellent job at the technical level: the resolution of the scanned pages is rather low so that even at first glance the types appear fuzzy and frayed. For a book that comes with a price tag of US-$ 129.00 / EUR 109.95 this is not tolerable.

Just as Chung's original text, the additional chapters of the new edition have grown from several sets of lecture notes. The original plan was to re-write Chung's classic and to weave in new material on time reversal of stochastic processes. Time reversal and the accompanying duality theory is best studied within the framework of Ray processes or even the *théorie générale*—topics which K.L. Chung carefully avoided in his 1982 monograph. In fact, Chung's old book is the attempt to teach the beautiful subject of Markov processes at an advanced level without running into the technicalities of the 'general theory'; it is part of its charm that this aim is achieved, resulting in a book which is easily accessible even to the novice. Facing these difficulties, the authors decided to reproduce the old text and to augment it with 10 entirely new chapters which are, strictly speaking, a book on their own. The focus is on time reversal and duality for Markov processes and, very much in the spirit of the first edition, the authors concentrate on the smallest reasonable class of Markov processes which still admit a nice duality theory—Ray processes—thus avoiding the depths of the 'general theory'.

Consequently, the new chapters treat a larger class of Markov processes and it happens that material is covered which is, for a narrower class of Markov processes, already known to the reader. The proofs are, however, quite different from the proofs of the first part.

The new chapters begin with the *Generalities* (§6), containing a few conventions, some notation—unfortunately the notation of part II differs sometimes from the notation used in part I, and this is not due to advances in the field—and a few unavoidable facts from the general theory of Markov processes; no proofs are given, the reader is referred to Dellacherie-Meyer. The three main chapters are §8 *Ray processes*, §10 *Time reversal* and §13 *Processes in duality*. The chapter on Ray processes gives a gentle introduction to the topic with emphasis on the Ray-Knight compactification. This allows to embed essentially every Markov process into a Ray process. Consequently, only time reversals of Ray processes are considered in §10. The central questions are: (i) when are the transition probabilities of the reversed process stationary? — (ii) is the reverse a strong Markov process? —, and, (iii) what is the connection with duality? Chapter 10 treats (i) and (ii) following original work by Chung and Walsh [To reverse a Markov process, *Acta Math.* **123** (1969), 225-51]. Problem (iii) is addressed in Chapter 13 where processes in duality and some elements of their potential theory are considered. Other topics covered are h-transforms (§11) and a probabilistic treatment of the Martin boundary (§14) which is essentially a Ray-Knight compactification of the dual process.

Sprinkled in between these chapters are three sections (7, 12 and 15) called *fireside chats*. These contain informally presented additional material; the intention is to give some mathematical intuition and understanding rather than rigorous proofs; nevertheless all claims and results are carefully stated. Topics getting this treatment are: Markov chains in continuous time, births and deaths of processes and duality measures.

Adding to Chung's masterpiece is a formidable task; the new chapters by Walsh capture the spirit of the original and give a gentle, inspiring and eminently useful introduction to Ray processes, time reversal and duality. Walsh's contribution is, however, also independent of the original text and it is a valid query why the authors decided to publish two books in one. The obvious alternative, to re-edit Chung's book, say, in Springer's *Classics in Mathematics* series and to publish Walsh's tract separately would have been possible, too. This, at least, would have circumvented some typographical inconsistencies within a single volume—the font, the numbering system and the layout of the new chapters differ considerably from the reproduced 1982 text. With some effort, LaTeX could have been adapted to produce a consistent appearance—but it seems that the publisher failed (again!) to do a

professionally satisfactory technical job.

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