To appear in: The Mathematical Gazette

Bourbaki, Nicolas: Integration I (Chapters 1-6) and II (Chapters 7-9). 2 volumes, Springer-Verlag, Berlin, Heidelberg 2004, xv + 472 and vii + 326 pp., £77.00 each, ISBN 0-540-41129-1 and 0-540-20585-3.

Nicolas Bourbaki is the pseudonym for a group of mathematicians who set out to create a *new mathematics*. Bourbaki was 'born' in Paris in 1935 when a small group of mathematicians at the *Ecole Normale Supérieure*, dissatisfied with the courses they were teaching, decided to reformulate them. Originally they had the goal to write an analysis textbook replacing the old-fashioned standard treatise by Goursat. Soon it turned out that this idea was quite naive and that a much more thorough approach was needed. This evolved into the plan to write the *éléments de mathématique* with six logically ordered core topics, set theory – algebra – topology – functions of one real variable – topological vector spaces – (Lebesgue) integration, which would be the foundation for all other more specialized topics. These elementary volumes were published from 1939–1970, with a break during World War II; in the intended second series only the volumes on commutative algebra, Lie groups and spectral theory (1983, Bourbaki's last book publication) ever appeared.

The books on integration were published in 1952 (Chapters 1-4, second edition in 1965), 1957 (Chapter 5, second edition in 1967), 1960 (Chapter 6), 1963 (Chapters 7,8) and 1969 (Chapter 9). The present edition in two volumes is the English translation (by S. K. Berberian) of the latest French edition. Volume 1 comprises Chapters 1-6, volume 2 Chapters 7-9 of the original monographs; the text itself is a mere translation, close to the original, without comments or extended updates. Unfortunately each volume is treated as a separate entity with its own list of references, notations and index. As in the original, the pages are numbered within each chapter only; no overall page numbers exist which is somewhat cumbersome when working with the text.

Bourbaki's approach to measure and integration is a functional analytic one. A Radon 'measure' (this notion was coined by Bourbaki) is a (possibly complex-valued) continuous linear functional acting on the compactly supported continuous functions over a locally compact space. This means that Bourbaki starts with the notion of an integral and develops in Chapters 1-4 the whole apparatus of integration theory up to L^p spaces. Parallel to it (set in small print in the French edition) the notion of an 'abstract' measure, i.e. a set-function, is developed and the connection to Radon measures is discussed. The price to pay for this mathematically elegant approach is a rather complicated notion of measurability. Chapter 5 treats *integrals of measures* and it is here where Bourbaki gives a unified framework for image measures, Radon-Nikodým derivatives, Lebesgue decompositions, etc. Curiously, the related topic of disintegration of measures is only treated in Chapter 6. Otherwise, Chapter 6 is on vector measures, both the integration of vector-valued functions w.r.t. a scalar measure and vector-valued measures, Chapter 7 introduces Haar measures and integrals on groups, and Chapter 8 treats convolutions and representations on groups. Originally this was all that was planned. Only later and *from an entirely different direction ... [arose] the need ... [of] considering measures on non locally compact topological spaces* (vol. 2, page IX.121). It was probability theory that showed that Bourbaki's point of view of Radon integrals on locally compact spaces was inadequate; as a reaction Chapter 9 was written as a general 'fix' and to cater for the needs of probability theory.

Bourbaki's treatise was the first which embedded measure and integration into functional analysis, taking up F. Riesz's approach. Parallel to it there was the development of an 'abstract' or 'algebraic' (Carathéodory) theory of measure and integration focussing more on set-functions. We know now (due to work by H. Bauer, *Bull. Soc. Math. France* **85** (1957), 51–75) that both theories are essentially equivalent. The Bourbaki volumes on integration are nowadays mainly of historic value, which is partly due to the (dogmatic) misconception which limited the theory to a locally compact setting and the, especially for probabilists, cumbersome notion of measurability. The chapter on vector measures is outdated by new developments (see Diestel and Uhl *Vector Measures*, Math. Surveys and Monographs vol. 15, Am. Math. Soc., Providence (RI) 1977, for a modern account), only the treatment of Haar measure and integration on groups retains its definitive character.

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