Review commissioned by: *Zentralblatt ZMath*

Jean Bertoin: Random fragmentation and coagulation processes. Cambridge University Press, Cambridge studies in advanced mathematics vol. **102**, Cambridge 2006, vii + 280 pp., £35.00, ISBN 0-521-86728-2.

Many natural phenomena involve spontaneous splitting or mergers, often in various forms and at various scales. Examples for fragmentation include mechanical grinding processes, breaking-up of DNA sequences in biology or degradation of polymer chains in chemistry; some, but not all of these processes, are reversible which then would be termed coagulation or (random) clumping. The aim of the present monograph is to provide a mathematical foundation and to develop models that can be used to describe such phenomena. The text is largely based on research papers, many contributions are due to the author himself and his collaborators and students.

The first three of the monograph's five chapters (pp. 1-160) deal with fragmentation. In order to get mathematically tractable models the author makes the following, restrictive but commonly accepted, assumptions: the fragmentation process is Markovian (i.e. memoryless), only the size of the particles is recorded while the spatial position and other geometric properties of the fragments are neglected, and the break-ups happen independently of each other and of the environment. These assumptions destroy, in general, the naive duality (by reversing the direction of time) of fragmentation and coagulation. This the reason why coagulation is treated separately in the second half of the book.

In Chapter 1 the author develops the theory of fragmentation chains which describe spontaneous splitting of the particles at discrete random times. Central notions are the genealogical structure of the chain (in the form of a marked random tree) and the branching property. The main results are asymptotic properties of the empirical measure of the fragments as time tends to infinity. Situations where splitting occurs continuously are studied in Chapter 3. Chapter 2 prepares the ground for this. The key notion is that of (random) mass partitions and their description in terms of Poisson random measures. It is here that the notion of exchangeable random partitions is introduced which can be used to encode mass partitions by random partition of the natural numbers (Kingman's paint-box construction). Exchangeable random partitions are used in Chapter 3 to construct general self-similar fragmentation processes which may split immediately. Chapter 3 also gives a characterization of the dynamics of the fragmentation process — Bertoin introduces an 'erosion' coefficient, a 'dislocation' rate and uses the index of self similarity — and studies the evolution of a fragment containing a randomly tagged point.

The second half of the book, Chapters 4 and 5 (pages 160-260), is devoted to exchangeable coalescents (Chapter 4) where the rates of coagulation are independent of the masses in the system and stochastic coalescents (Chapter 5) where only two particles are allowed to merge but where the rate may depend on the masses of the fragments. Chapter 4 begins with the study of Kingman's coalescent (only binary coagulations) and then discusses more general coalescents which allow several fragments to merge simultaneously. The chapter closes with some recently discovered connections to stochastic flows. In the last chapter the asymptotic properties of Marcus-and-Lyshnikov's stochastic coalescent is studied. The focus is on the Feller property of the transition functions and the hydrodynamic behaviour of some stochastic coalescents and their relation to Smoluchwski's equation.

This monograph is an authoritative and important contribution to the research literature. Most of the material appears for the first time in the form of a monograph. The exposition is both concise and lucid, but clearly pitched at (post-)graduate level; a sound background in probability theory is a *sine qua non* and the reader should be familiar with (a bit more than just the basics of) stochastic processes, Markov chains, Poisson random measures etc., since these tools are just briefly introduced — basically to fix the notation. Bertoin does not try to be encyclopaedic but concentrates on some essentials. This approach makes the text very accessible; detailed comments following each chapter and an extensive bibliography comprising more than 200 entries make up for any losses caused by the the author's selection.

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