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**Richard Bass:** *Stochastic Processes.* Cambridge University Press, Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge 2011, xvi + 390 pp., £45.00, US-$ 75.00 ISBN 978-1-107-00800-7.

Stochastic processes in continuous time are a central part of a probabilist's curriculum. The way we teach the subject changes from time to time, sometimes dictated by fashions—jumps vs. diffusions, general theory vs. concrete cases—and quite often driven by the needs of applications such as mathematical finance. Over the past two decades we have seen specialized monographs on Brownian motion and diffusions, Lévy processes, Gaussian processes, Markov processes, some of them written as introductions to stochastic processes in general and the processes mentioned on the frontpage in particular, some of them clearly for the 'adult' reader. The book under review continues a different tradition as it sets out to give a general, though not an abstract introduction to stochastic processes. In fact, almost all important classes of elementary stochastic processes are touched upon—Brownian motion, (continuous semi-)martingales, additive functionals, Markov processes, Poisson and Poisson point processes, Gaussian processes and Lévy processes—and both the probabilists' path-by-path approach and the analysts' semigroup and evolution equation treatment are covered. At first sight this seems to be impossible to achieve on a mere 390 pages but a closer look at the text will convince us of the contrary.

There are 42 rather short chapters in total which need not be read linearly. Clearly, this is way too much material for a semester- or year-long course, especially since the presentation is sometimes a bit condensed. Some ideas how to use the book are given in the preface. The core is an introduction to Brownian motion and some basic path properties (in Chapters 1–8) and stochastic calculus for continuous (semi-)martingales (Chapters 9–12). The aim (as everywhere else in the book) is not to get wound up in technicalities but to present a clear picture of things; of course, this comes at a price, and some results are not presented in their greatest generality (e.g. the Burkholder–Davis–Gundy theorem is 'only' shown for $p \geq 2$ and the Dambis–Dubins–Schwarz theorem is given only in its simplest variant) but usually the balance between clarity and technicality is very well kept. This core material is dealt with on the first 80 pages. From now on there are plenty of choices:

- one could go to applications in finance (Chapter 28) and engineering (filtering, Chapter 29), via chapters 13 (Girsanov), 14 (local times), 15 (Skorokhod embedding) and 24–25 (SDEs)
- if one is interested in jump processes, the route would lead through Chapter 16 (general theory of processes), 17, 18 (jump and Poisson point processes), 30 (weak convergence), 34 (Skorokhod space) to Lévy processes (Chapter 42)
- one may study Markov processes and their transformations (Chapters 19–22), applications to optimal stopping (Chapter 23) and a prelude to analytic techniques: semigroups, generators, Dirichlet forms and SDEs (Chapters 36–39).
- one can continue studying Brownian motion and diffusions in depth: Local times and Skorokhod embedding (Chapters 14, 15), Ray-Knight theorems and excursions (Chapter 26, 27), weak convergence in $C[0,1]$ (Chapters 30, 32) and Donsker's theorem (Chapter 35) leading to applications in PDEs (Chapter 40) and Feller diffusions (Chapter 41).

Some of the material I have never seen in a textbook at this (intermediate) level: filtering, Skorokhod space, Dirichlet forms and Feller diffusions are certainly a welcome addition to this type of text.

Ideally, the reader should have some background knowledge of classical measure-theoretic probability theory and discrete-time martingales; the books *Probability with Martingales* by David Williams [MR1155402 (93d:60002)] or *Probability Essentials* by Jean Jacod and Philip Protter [MR1736066 (2001b:60001)] provide about the right level for this. Otherwise the text is self-contained, and none of the core material is shifted to the DIY problems sections (although there are more than 350 problems which are not just for drill).

The text would be even more valuable if the author had included more references and guidance to further reading. Variations of the rather laconic sentence 'See reference … for further information' which appear at the end of many later chapters is not really helpful; without further guidance, the monographs referred to are partly out of scope for the typical reader of this textbook. A few more words could change this.

Despite this little criticism, this is a great book which helps the graduate student to get a taste of stochastic processes—and, I am sure, a good appetite, too. For instructors it is a valuable source of new topics for their next lecture course.

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