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Gernot Akemann, Jinho Baik, Philippe Di Francesco (eds): ***The Oxford Handbook of Random Matrix Theory***. Oxford University Press: Oxford, New York 2011. Xxxi + 919 pp., £110.00, \$ 198.50, €140.99 (RRP), ISBN: 978-0-19-957400-1.

Random matrices appeared for the first time around the 1930s in mathematical statistics and, later on, in the 1950s in connection with statistical models for heavy-nuclei atoms. In its basic form, a random matrix is a (often fairly high-dimensional) matrix whose entries are real or complex random variables. Physicists use random matrices as (finite-dimensional, coordinate) approximations of Hamiltonians; if the Hamiltonian itself is unknown or too complicated, one makes statistical hypotheses which reflect the general symmetry properties—this is how randomness enters. In principle, any physical information can be deduced from the eigenvalues and eigenfunctions. Therefore, typical questions concern the distribution of these quantities. Wigner's 1955 Annals of Mathematics paper[1] is nowadays recognized as the starting point of modern random matrix theory (for short: RMT). Here he proves his 'semi-circle law' which says that the empirical distribution of the eigenvalues approaches weakly the probability law $\frac{1}{2\pi}\sqrt{4-x^2}\,\mathbf{1}_{\{|x|\leqslant 2\}}\,dx$, as the dimension of the matrix tends to infinity. The limiting law is universal in the sense that it does not depend on the laws of the original random variables but only on the symmetry structure of the underlying random matrix (nowadays called Wigner matrices).

Starting from these applied roots, RMT has been intensively studied in applications and it has even found its way into the purest of all mathematical subjects: number theory. The handbook gives a short historical introduction (Part 1) to RMT and develops in the 21 subsequent chapters physical (e.g. symmetry and supersymmetry, ensemble analysis, universality, criticality, phase transitions) and mathematical (integrable systems, determinantal processes, Wigner matrices, free probability) aspects of the theory of random matrices. The third part of the handbook is devoted to applications of RMT. Having in mind the 'physical' conception of RMT, the connection with the Riemann hypothesis (*all non-trivial zeroes of the zeta function are on the critical line* $\frac{1}{2}+\mathrm{i}t$) is most striking. Many more pure and applied

[1] E.P. Wigner: Characteristic vectors of bordered matrices with infinite dimensions. *Ann. Math.* **62** (1955) 548–564.

subjects witness the effectiveness of RMT techniques: enumeration theory, knots, multivariate statistics, algebraic geometry, quantum gravity, strings, chaos theory...—to mention less than half of the 20 topics addressed in the applications part of the monograph.

The contributors to this volume are research-active specialists from physics and mathematics. Each chapter is an essentially self-contained presentation of the topic at hand, but there are also cross-references to other contributions within the volume. Most of the contributions are written in a style which is accessible to a general mathematical or physical audience and, more importantly, which encourages the interested reader to consult some of the references to the original literature. For the novice the first two chapters are a well-written and helpful invitation to RMT.

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