René L. Schilling: **Measures, Integrals, and Martingales (2nd edn)**

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Misprints and smaller changes. Updated: February 3, 2026.

Page, Line	Reads	Should Read
p. 5, Problem 1.5	$S_0, S_2, S_2, \dots$; *and, next line:* $S_\infty := \dots$	$S_0, S_1, S_2, \dots$; $S := \dots$
p. 21, Prob. 3.6	**Remark.** A set $A \dots$	**Remark.** A set $A \neq \emptyset \dots$
p. 22, Prob. 3.14	which contains X	which contains $\emptyset$, X
p. 25, line 1 above	$A \cup B = (A \uplus \dots$	$A \cup B = A \uplus \dots$ (*remove leading brace*)
p. 28, Lemmas 4.8, 4.9	measure space (*twice*)	measurable space (*twice*)
p. 29, Prob. 4.6	assigns to every interval $[a, b)$ with $b - a > 2$ finite mass	assigns to every interval $[a, b)$ with $b - a > 2$ infinite mass
p. 29, Prob. 4.8	finitely additive	is finitely additive
p. 38, Prob. 5.13	which contains X	which contains $\emptyset$, X
p. 38, Prob. 5.13(i)	formation of complements.	formation of complements and finite intersections.
p. 43, line 4 below	$\mu(S_n)$	$\mu(S_i)$
p. 47, line 1,2 above	On the other hand, monotonicity ... entail ... $\leq \lambda[a, b)$,	On the other hand, for each I_n, $n = 1, \dots, N$ there is some $I'_n \in \mathscr{J}$ such that $I_n \subset I'_n$ and $\biguplus_{n=1}^N I'_n = [a, b)$. Monotonicity and finite additivity of λ entail $\sum_{n=1}^N \lambda(I_n) \leq \sum_{n=1}^N \lambda(I'_n) = \lambda\left(\biguplus_{n=1}^N I'_n\right) = \lambda[a, b)$,
p. 50, Prob. 6.1(ii)	$\forall a, b \in \mathbb{R}$, $a < b$	$\forall a, b \in \mathbb{R}$, $a \leq b$
p. 50, Prob. 6.4	Recall from Problem 9.14	Recall from Problem 4.15
p. 54, line 2 above	Theorem 5.6	Theorem 5.8
p. 57, line 5 below	linear	affine linear
p. 58, line 4 above	$\tau_x(\lambda^n) \overset{7.10}{=} \lambda^n$	$\tau_x(\lambda^n) \overset{7.8}{=} \lambda^n$
p. 59, Prob. 7.12	$C_2 = J_2^{00} \uplus J_2^{01} \uplus J_2^{10} \uplus I_2^{11}$	$C_2 = J_2^{00} \uplus J_2^{01} \uplus J_2^{10} \uplus J_2^{11}$
p. 62, line 5 below	$\sum_{j=m}^M$	$\sum_{j=0}^M$ *where:* $A_0 := (A_1 \cup \dots \cup A_M)^c$
p. 69, line 3 above	$\mathbb{1}_{\biguplus_{n\in\mathbb{N}} \mathbb{1}_{A_n}}$	$\mathbb{1}_{\biguplus_{n\in\mathbb{N}} A_n}$
p. 79, Problem 9.5	$\int u_n \, d\mu \uparrow \int u \, d\mu$	$\int u_{n+K} \, d\mu \uparrow \int u \, d\mu$
p. 88, Prob. 10.9.(i)	$\{g^{-1}(B) : \mathscr{B}(\mathbb{R}^2)\}$	$\{g^{-1}(B) : B \in \mathscr{B}(\mathbb{R}^2)\}$
p. 91, line 12 above	Corollary 11.3	Theorem 10.3(iv) together with Corollary 11.3
p. 92, line 8 below	Theorem 9.6(i)	Theorem 11.2(i)
p. 93, line 6	Corollary 11.7(iii)	Corollary 11.7(ii)
p. 93, Prob. 11.3(vi)	$\mathbb{V}\xi = \int (\xi - \mathbb{E}\xi)^2 \, dP$	$\mathbb{V}\xi = \int (\xi - \mathbb{E}\xi)^2 \, d\mathbb{P}$
p. 98, line 1/2 above	Corollary 11.4(iv)	Corollary 11.4
p. 105, line 8 above	$x \in D$	$x \notin D$
p. 105, line 10 below	Appendix G	Appendix E
p. 120, line 7 above	$\frac{1}{n^p}$	$\frac{1}{n}$
p. 126, line 9 below	$V(\infty) := +\infty$	$V(\infty) := \lim_{x\to\infty} V(x) \in [-\infty, +\infty]$ Remark: By convexity, V is “finally” monotone.
p. 126, line 4 below	$u \in \mathcal{L}^1(\mu)$, $u \geq 0$, implies	$u \in \mathcal{L}^1(\mu)$, $u \geq 0$, $V(\infty) = \infty$, implies

turn over

Page, Line	Reads	Should Read
p. 127, line 2 above	$+\infty \mathbb{1}_{u=\infty}(x)$	$+V(\infty)\mathbb{1}_{u=\infty}(x)$
p. 137, line 6 above	und	and
p. 147, line 8 above	Theorem 12.9(ii)	Theorem 12.9
p. 151, Prob. 14.13	$u : \mathbb{R}^2 \to [0, \infty]$	$u : \mathbb{R} \to [0, \infty]$
p. 153, Prob. 14.19(ii)	$\int_{\mathbb{R}} \cdots$	$\int_X \cdots$
p. 168/9, Proof of L. 16.7, T. 16.4	$\mathscr{J}$ (*all instances*)	$\mathscr{J}_{\mathrm{rat}}$ (*all instances*, i.e. rectangles with rational end-points, see p. 19)
p. 171, line 3 above	... such that	... with rational end-points such that
p. 185, Ex. 16.11	Euler's Inegrals.	Euler's Integrals
p. 187, Caution	Lemma 17.3 does not hold for $p = \infty$	Lemma 17.2 does not hold for $p = \infty$ if we want that the supports of the simple functions have finite μ-measure.
p. 190, Thm. 17.8	σ-compact	locally compact and σ-compact Remark: one needs that $K_n \subset K_{n+1}^{\circ} \subset K_{n+1} \uparrow X$, see also Prob. 17.6
p. 192, line 13 above	compact sets	open balls
p. 194, line 11 above	$\Vert u\Vert_\infty \mathbb{1}_{\mathrm{supp}\, u}(x) e^{-\vert z\vert^2/2}$	$\Vert u\Vert_\infty \mathbb{1}_{\mathbb{R}^n}(x) e^{-\vert z\vert^2/2}$
p. 200, line 12 below	$\sum_{i=1}^\infty \mu_\epsilon^*(A_i) \leq \mu^*(A)$	$\sum_{i=1}^\infty \mu_\epsilon^*(A_i) \leq \sum_{i=1}^\infty \mu^*(A_i)$
p. 208, line 5 below	$\pi^{n/2}/\Gamma\left(\frac{1}{2}+1\right)$	$\pi^{n/2}/\Gamma\left(\frac{n}{2}+1\right)$
p. 236, line 4 above	$\int \vert\mathbb{I}_A - u\vert^2 \, d\nu$	$\int \vert\mathbb{I}_A - u\vert \, d\nu$
p. 237, lines 11/12 above	$(0 \vee f \wedge 1)^2 = 0 \vee f^2 \wedge 1$ $(0 \vee (1-f) \wedge 1)^2 = 0 \vee (1-f)^2 \wedge 1$	$(0 \vee f \wedge 1)^2 \leq 0 \vee f^2 \wedge 1$ $(0 \vee (1-f) \wedge 1)^2 \leq 0 \vee (1-f)^2 \wedge 1$
p. 238, line 14 above	$\left\{u \in C(X) : \lim_{\vert x\vert\to\infty} u(x) = 0\right\}$	$\left\{u \in C(X) : \forall \epsilon > 0 \; \exists K \subset X \text{ compact } \forall x \in K^c : \vert u(x)\vert \leq \epsilon\right\}$
p. 241, line 10 below	*add the following sentence →*	If $p = 1$ and $q = \infty$ we use $\mathrm{sgn}(h)\mathbb{I}_{A_n}$ (A_n is an exhausting sequence) instead of $\mathrm{sgn}(h)\vert h\vert^q$. This gives $h\mathbb{I}_{A_n} = 0$ a.e., hence $f = g$ a.e.
p. 243, line 12 below	Yosida [59, Section IV.9, Example 3]	Dunford and Schwartz [15, Section IV.8.1]
p. 243, line 8 below	Rheorem	Theorem
p. 252, line 7 above	In particular,	If X is separable,
p. 267, line 9 below	$\int_{B^+} u \, d\mu - \int_{B^-} (-u) \, d\mu$	$\int_{B^+} u \, d\mu + \int_{B^-} (-u) \, d\mu$
p. 270, line 5 above	$\int_{K_{n(k)}^c} (-u_{k+1}^-) \, d\mu$	$\int_{K_{n(k)}^c} u_{k+1}^- \, d\mu$
p. 299, hint 24.9(iii)	with $\tau_\kappa := \inf\{n : \vert M_n\vert > \kappa\}$ ... $M_{\tau\wedge n_\kappa}$ ... $A_{\tau\wedge n_\kappa}$	with $\tau_\kappa := \inf\{n : \vert S_n\vert > \kappa\}$ ... $S_{\tau_\kappa\wedge n}$ ... $A_{\tau_\kappa\wedge n}$
p. 314, line 13 above	$\mu(Q)/\lambda^n(Q)$	$\mu(e+Q)/\lambda^n(Q)$
p. 334, line 11 below	$\Vert g - P_E g\Vert$	$\Vert g - P_E g\Vert^2$
p. 362, line 11 above	Theorem 27.19	Corollary 27.20
p. 367, Prob. 27.8	$\mathbb{E}^{\mathscr{G}} = \mathbb{E}^{\mathscr{G}}$	$\mathbb{E}^{\mathscr{G}} = E^{\mathscr{G}}$
p. 385, line 4 above	$\{\chi_{k,j} = \pm 1\}$	$\{\chi_{k,j} = \pm 2^{k/2}\}$
p. 389, line 15 above	Note that $\psi = \psi_{0,1} = \chi_{0,1}$ ... while $\psi_{0,-1}(x) =$ ...	Note that $\psi = \psi_{0,0} = \chi_{0,1}$ ... while $\psi_{-1,0}(x) =$ ...
p. 417, line 16 below	$_K \leq u_\epsilon \leq \mathbf{1}_{U_\epsilon}$	$\mathbb{1}_K \leq u_\epsilon \leq \mathbb{1}_{U_\epsilon}$
p. 417, line 14 below	$0 \leq w_\epsilon \leq \; _U$	$0 \leq w_\epsilon \leq \mathbb{1}_U$
p. 422, line 3 below	$\ldots \overset{C.3}{=} \ldots$	$\ldots \overset{7.9}{=} \ldots$
p. 442, line 10 above	Obviously ... they satisfy	If $\vert u(x)\vert \leq M$, then they satisfy
		turn over

Page, Line	Reads	Should Read
p. 442, line 11 above	$\lvert S_\pi[u]\rvert \le S_\pi[\lvert u\rvert] \le M(b-a)$ $\lvert S^\pi[u]\rvert \le S^\pi[\lvert u\rvert] \le M(b-a)$	$\lvert S_\pi[u]\rvert \le M(b-a)$ $\lvert S^\pi[u]\rvert \le M(b-a)$
p. 442, line 2 below	$\inf_\pi S_\pi[u]$	$\inf_\pi S^\pi[u]$
p. 465–467 References	*concerns: page numbers (given in parentheses at the end of each entry) where references are used*	*by some error, you have to add $n \in \{2,3,4,5\}$ to the page numbers given*

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Correction of the proof of Proposition 6.6

The proof of Proposition 6.6 on page 48/49 is incorrect and must be changed as follows.

Proof of Proposition 6.6. We have to show that $\lambda^n(\emptyset) = 0$ and that λ^n is σ-additive on $\mathcal{J}^n$. It is clear that $\lambda^n(\emptyset) = 0$. To see σ-additivity, we use induction with respect to the dimension n. Proposition 6.3 covers the case $n = 1$. We assume that $\nu = \lambda^n$ is σ-additive on the rectangles $\mathcal{J}^n$ for some $n \ge 1$.

Step 0: In the proof we use repeatedly the fact that sets of the form $I \setminus (J \cup K)$, $I, J, K \in \mathcal{J}^n$, can be written as $I \setminus (J \cup K) = I \cap J^c \cap K^c = (I \setminus J) \setminus K$. In view of property (S3) of a semiring, each difference can be represented as a finite union of disjoint rectangles and, thus, $I \setminus (J \cup K)$ is a finite union of disjoint rectangles from $\mathcal{J}^n$.

Step 1: We first show finite additivity and sub-additivity of λ^{n+1}. Let $J_1, \dots, J_N \in \mathcal{J}^{n+1}$ such that

$$I = \bigcup_{j=1}^N J_j = I^n \times I^1 \in \mathcal{J}^n \times \mathcal{J}^1 = \mathcal{J}^{n+1}.$$

We can write I also as a union of disjoint sets from $\mathcal{J}^{n+1}$. To do so, consider $J_1, J_2 \setminus J_1, J_3 \setminus (J_2 \cup J_1), \dots, J_N \setminus (J_{N-1} \cup \dots \cup J_1)$. Applying to each of these sets Step 0, we find mutually disjoint sets $I_i = I_i^n \times I_i^1 \in \mathcal{J}^{n+1}$, $i = 1, \dots, M$, such that

$$I = \bigcup_{j=1}^N J_j = \dot{\bigcup_{i=1}^M} I_i = \left(\bigcup_{i=1}^M I_i^n\right) \times \left(\bigcup_{i=1}^M I_i^1\right) = I^n \times I^1 \in \mathcal{J}^{n+1}.$$

We want to represent also the (cartesian) factors as unions of disjoint sets. Fix $d \in \{1, n\}$. Define for $k = 1, 2, \dots, 2^M$ the sets $\hat{I}_1^d \cap \hat{I}_2^d \cap \dots \cap \hat{I}_M^d \cap I^d$, where $\hat{I}_i^d$ is either I_i^d or $(I_i^d)^c$, making the sets mutually disjoint (note that some of them might be $\emptyset$). By Step 0, each of these sets can be represented as a finite union of disjoint rectangles from $\mathcal{J}^d$. In total, we have for suitable $K_k^d \in \mathcal{J}^d$ and $d \in \{1, n\}$

$$I = \left(\dot{\bigcup_{k=1}^P} K_k^n\right) \times \left(\dot{\bigcup_{l=1}^Q} K_l^1\right) = \dot{\bigcup_{i=1}^M} \overbrace{\dot{\bigcup_{(k,l):\,K_k^n \times K_l^1 \subset I_i}} K_k^n \times K_l^1}^{=I_i} \tag{*}$$

$$= \bigcup_{j=1}^N \overbrace{\dot{\bigcup_{i:\,I_i \subset J_j}} \dot{\bigcup_{(k,l):\,K_k^n \times K_l^1 \subset I_i}} K_k^n \times K_l^1}^{=J_j}. \tag{\#}$$

Thus,

$$\lambda^{n+1}(I) = \lambda^n\left(\dot{\bigcup_{k=1}^P} K_k^n\right) \lambda^1\left(\dot{\bigcup_{l=1}^Q} K_l^1\right) = \sum_{k=1}^P \sum_{l=1}^Q \lambda^n\left(K_k^n\right) \lambda^1\left(K_l^1\right) = \sum_{k=1}^P \sum_{l=1}^Q \lambda^{n+1}\left(K_k^n \times K_l^1\right).$$

Now we can use (*) and (#), and rearrange the double sum to obtain finite (sub-)additivity:

$$\lambda^{n+1}(I) = \sum_{i=1}^{M} \lambda^{n+1}(I_i) \leq \sum_{j=1}^{N} \lambda^{n+1}(J_j).$$

Step 2: We assume now that $(I_i)_{i\in\mathbb{N}} \subset \mathcal{J}^{n+1}$ are mutually disjoint rectangles $I_i = I_i^n \times I_i^1$ such that

$$I = \bigcupdot_{i\in\mathbb{N}} I_i = I^n \times I^1 \in \mathcal{J}^n \times \mathcal{J}^1 = \mathcal{J}^{n+1}.$$

For every $\epsilon > 0$ we may circumscribe around each I_i a rectangle $I_{i,\epsilon} \in \mathcal{J}^{n+1}$ such that the closure of I_i is in the open interior of $I_{i,\epsilon}$:

$$\overset{\circ}{I}_{i,\epsilon} \supset \overline{I_i} \quad \text{and} \quad \lambda^{n+1}(I_{i,\epsilon}) - \lambda^{n+1}(I_i) \leq \frac{\epsilon}{2^i}.$$

Clearly, $\bigcup_{i\in\mathbb{N}} \overset{\circ}{I}_{i,\epsilon} \supset I \supset \overline{I^\epsilon}$ where $\overline{I^\epsilon}$ is a closed rectangle inscribed into I, such that $\lambda^{n+1}(I) - \lambda^{n+1}(\overline{I^\epsilon}) \leq \epsilon$. Since $\overline{I^\epsilon}$ is compact, there is some finite sub-cover, i.e. some $N \in \mathbb{N}$ such that

$$I^\epsilon = \left(\bigcup_{i=1}^{N} I_{i,\epsilon} \right) \cap I^\epsilon = \bigcup_{i=1}^{N} \underbrace{\left(I_{i,\epsilon} \cap I^\epsilon \right)}_{\in \mathcal{J}^{n+1}}.$$

Therefore,

$$\lambda^{n+1}(I) - \epsilon \leq \lambda^{n+1}(I^\epsilon) \overset{\text{Step 1}}{\leq} \sum_{i=1}^{N} \lambda^{n+1}(I_{i,\epsilon} \cap I^\epsilon) \leq \sum_{i=1}^{N} \lambda^{n+1}(I_{i,\epsilon}) \leq \sum_{i=1}^{N} \lambda^{n+1}(I_i) + \frac{\epsilon}{2^i} \leq \epsilon + \sum_{i=1}^{\infty} \lambda^{n+1}(I_i).$$

Using repeatedly (S3), we see that $I \setminus \bigcupdot_{i=1}^{N} I_i = I \cap I_1^c \cap \dots \cap I_N^c = \bigcupdot_{k=1}^{M} R_k$ for mutually disjoint rectangles $R_k \in \mathcal{J}^{n+1}$, i.e. $I = I_1 \cupdot \ldots \cupdot I_N \cup R_1 \cupdot \ldots \cupdot R_M$. Using the finite additivity from Step 1 yields

$$\sum_{i=1}^{N} \lambda^{n+1}(I_i) \leq \sum_{i=1}^{N} \lambda^{n+1}(I_i) + \sum_{k=1}^{M} \lambda^{n+1}(R_k) = \lambda^{n+1}(I).$$

Letting $N \to \infty$ and $\epsilon \downarrow 0$ finally proves the claimed σ-additivity on $\mathcal{J}^{n+1}$. $\square$