

Index

- additive functional, 288
- analytic function of bounded type, 58
 - canonical factorization, 58
- Balakrishnan's formula, 145
- Bernstein function, 15, 132
 - characterization, 19, 142
 - complete,
 - see* complete Bernstein function
 - cone structure, 19
 - convergence of, 21
 - definition, 15
 - extended, 38, 60, 89
 - extremal representation, 24
 - is negative definite, 28, 31
 - Lévy measure, 15
 - Lévy triplet, 15
 - Lévy-Khintchine representation,
 - 15, 37, 43, 46, 49, 115, 135, 185
 - relation with \mathcal{CM} , 19, 35, 39
 - special,
 - see* special Bernstein function
 - stability, 19
- Bernstein's theorem, 3, 17, 29, 50
- Bochner's theorem, 32
- Bochner-Schoenberg theorem, 142
- Bondesson class, 80
 - and first passage time, 205
 - as vague closure, 83
 - definition, 80
 - stability, 80
 - $\text{BO} \subset \text{ID}$, 80
 - $\text{BO} \not\subset \text{SD}$, 91
- capacity, 161, 287
- Choquet representation
 - for \mathcal{BF} , 24
 - for \mathcal{CBF} , 65
- for \mathcal{CM} , 7
- for \mathcal{S} , 13
- for \mathcal{TBF} , 78
- for positive definite fns, 29–31
- complete Bernstein function, 49
 - and inverse local time, 186
 - characterization, 49–50, 62, 63, 66, 67, 112
 - cone structure, 64
 - conjugate pair, 62
 - convergence of, 65
 - definition, 49
 - exponential representation, 59, 67, 68, 82, 87
 - extremal representation, 65
 - is Nevanlinna-Pick function, 49
 - is operator monotone, 121
 - is Pick function, 49
 - Lévy density vs. Stieltjes measure, 58
 - log-convexity of \mathcal{CBF} , 67, 68, 70
 - multiplicative representation, 60
 - Nevanlinna representation, 57, 70
 - potential density, 95
 - relation between Lévy density and Stieltjes measure, 164
 - relation with \mathcal{S} , 63, 66–67
 - stability, 63, 64, 71, 81
 - Stieltjes measure, 55
 - Stieltjes representation, 55, 62, 63, 65, 69, 80, 112, 116
 - $\mathcal{CBF} \subset \mathcal{SBF}$, 92
- completely monotone function, 2
 - cone structure, 5
 - convergence of, 6
 - definition, 2
 - extremal representation, 7
 - infinitely divisible, 37

- is positive definite, 28, 31
 - iterated differences, 29
 - logarithmically,
 - see* log-completely monotone representation, 3
 - self-decomposable, 41
 - stability, 5, 20, 43, 282
 - stable, 40
- convolution of exp. distributions, 87
 - and first passage time, 205, 209
 - $CE \subset GGC$, 88
- convolution semigroup, 34, 132, 135
 - characterization, 35
 - definition, 34
 - potential measure of, 44
 - subordinate, 46
- Dirichlet form, 161, 286
- dissipative operator, 111, 131
 - spectrum, 111
- distribution
 - \mathcal{S} -self-decomposable, 86
 - infinitely divisible, 37, 39
 - Laplace exponent of, 90
 - overview, 89
 - relations among, 90
 - self-decomposable, 41
 - stable, 40
- excessive function, 177
- Feller transition function, 285
- first passage time
 - and BO , 205
 - and CE , 205, 209
 - and GGC , 205
 - and ME , 209
 - generalized diffusion, 201
- fractional power, 91
 - Balakrishnan's formula, 145
 - in \mathcal{BF} , vi, 15, 24
 - in \mathcal{CBF} , vii, 65
 - in \mathcal{CM} , vi, 41
 - in \mathcal{S} , vii, 13
 - in \mathcal{JBF} , vii
 - of operators, 145
- functional calculus, 150
- generalized diffusion, 185
 - first passage time, 201
 - inverse local time, 186
 - local time, 185
- generalized Gamma convolution, 84
 - and first passage time, 205
 - as vague closure, 85
 - definition, 84
 - stability, 84
 - $GGC \subset BO$, 84
 - $GGC \subset SD$, 84
 - $GGC \not\subset ME$, 91
- harmonic function, 177
- Heinz-Kato inequality, 122
- Hellinger-Stone formula, 114
- Helly's selection theorem, 282
- Hirsch function, 105
 - characterization, 105
 - cone structure, 105
 - definition, 105
 - relation with \mathcal{P} , 107
 - relation with \mathcal{SBF} , 107
 - relation with \mathcal{S} , 107
 - stability, 105
- Hunt process, 160, 184, 284
- inconsistency
 - unavoidable, 1–305
- infinitely divisible,
 - see* distribution, random variable
- inverse local time
 - and \mathcal{CBF} , 186
 - generalized diffusion, 186
- killed Brownian motion, 176
- killed process, 161

- subordinate, 163
- Kreĭn correspondence, 200
 - examples, 200–201
- Kreĭn representation problem, 200
- Kreĭn-Milman representation,
 - see* Choquet representation
- Lévy process, 118, 142, 169
- Lévy triplet, 15
- Laplace exponent, 35
 - of distribution, 89
 - relations among, 90
- Laplace transform, 1
- Lévy-Khintchine formula, 32
- local time
 - generalized diffusion, 185
- log- \mathcal{CM} , 89
- log- \mathcal{S} , 89
- log-completely monotone function
 - characterization, 38
 - definition, 38
 - relation with \mathcal{P} , 45
- log-convex function, 97
- log-convex sequence, 97, 105
- Markov process, 283
 - definition, 283
 - generator, 285
 - normal, 283
 - quasi left-continuous, 283
 - resolvent kernel, 284
 - semigroup, 285
 - strong, 283
 - transition function, 284
- Markov property, 283
 - strong, 283
- matrix monotone function, 119
- measure
 - Lévy, 15, 214
 - Nevanlinna-Pick, 57
 - Pick, 57
 - Stieltjes, 55, 214
 - Thorin, 75, 214
- mixture of exponential distributions
 - and first passage time, 209
 - characterization, 81
 - definition, 81
 - stability, 83
 - $ME \subset BO$, 83
 - $ME \not\subset GGC$, 91
 - $ME \not\subset SD$, 91
- monotone matrix function, 119
- negative definite function, 26, 142
 - in the sense of Schoenberg, 32, 118, 142
- Nevanlinna factorization, 58
- Nevanlinna-Pick function, 56
- Nevanlinna-Pick measure, 57
- operator monotone function
 - definition, 119
 - is \mathcal{CBF} , 126
- Phillips' theorem, 135
- Pick function, 56
- portmanteau theorem, 281
- positive definite function, 25
 - conditionally, 26
 - in the sense of Bochner, 32
- potential, 45, 173
 - relation with \mathcal{H} , 107
- potential measure, 44
 - λ -potential measure, 45
- potential operator, 173
 - λ -potential operator, 186
- quasi everywhere, 161, 287
- random variable
 - infinitely divisible, 37, 39
 - stable, 40
- representing measure
 - for \mathcal{BF} , 15
 - for \mathcal{CBF} , 55

- for \mathcal{CM} , 4
 - for \mathcal{S} , 12
 - for \mathcal{JBF} , 75
- resolvent equation, 110, 131, 285
- Revuz measure, 288
- \mathcal{S} -self decomposable distribution
 - characterization, 86
 - definition, 86
 - is $\text{GGC} \cap \text{ME}$, 86
- Schoenberg's theorem, 26
- Schoenberg-Bochner theorem, 142
- self-adjoint operator
 - dissipative, 111, 131
 - order, 119
 - resolvent, 110, 131
 - resolvent equation, 110
 - resolvent estimate, 131
 - spectral theorem, 114
 - spectrum, 110, 131
- self-decomposable distribution, 41
 - Lévy measure of, 42
 - $\text{SD} \not\subset \text{BO}$, 90
 - $\text{SD} \not\subset \text{ME}$, 90
- semigroup,
 - see also* convolution semigroup,
 - see also* subordinate semigroup
 - algebraic, 25
 - C_0 -semigroup, 130
 - contraction semigroup, 130, 132
 - generator, 130, 285
 - intrinsically ultracontractive, 173, 177
 - strongly continuous, 130, 285
 - ultracontractive, 165
- smooth measure, 287
- special Bernstein function, 92, 97
 - cone structure, 103
 - conjugate Lévy triplet, 95
 - conjugate pair, 92
 - counterexample, 102–103
 - definition, 92
 - potential density, 94
 - relation with \mathcal{H} , 107
 - stability, 103
 - $\mathcal{SBF} \not\subset \mathcal{CBF}$, 102–103
- special functions, 215–217
- special subordinator, 92, 97, 173
 - characterisation, 93
 - factorization of potential density, 96, 174, 175
 - potential density, 94
- spectral theorem, 114, 120
- Stieltjes function, 11
 - and string, 189
 - cone structure, 12
 - definition, 11
 - extremal representation, 13
 - log-convexity of \mathcal{S} , 67
 - primitive, 70–71
 - relation with \mathcal{CBF} , 63, 66–67
 - relation with \mathcal{H} , 107
 - stability, 12, 64, 83
- Stieltjes inversion formula, 55
- Stieltjes transform,
 - see* Stieltjes function
- string, 184
 - and Stieltjes function, 189
 - characteristic function, 190
 - definition, 184
 - dual, 200
 - length, 184
- subordinate generator, 133
 - \mathcal{CBF} -subordinator, 149
 - domain, 138, 148, 156–158
 - functional calculus, 150–151
 - limits, 158
 - moment inequality, 139
 - operator core, 134
 - Phillips' formula, 135
 - spectrum, 141
- subordinate process, 46, 141, 162
 - killed, 163

- subordinate semigroup, 46, 132–133
 - definition, 133
 - in Hilbert space, 133
 - of measures, 46
- subordinator, 35, 141
 - killed, 36
 - transition probabilities, 35
- symmetric α -stable process, 169
- table
 - of Kreĭn correspondences, 200–201
 - of distributions, 89–90
 - of Laplace exponents, 89–90
- Thorin-Bernstein function, 73
 - characterization, 73, 76, 86
 - cone structure, 77
 - definition, 73
 - extremal representation, 78, 85
 - is in \mathcal{CBF} , 73
 - relation with \mathcal{S} , 77
 - Thorin measure, 75
 - Thorin representation, 75, 78, 85
- transition function, 35, 284
- vague convergence, 280
- vague integral, 281
- weak convergence, 280