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Symbols and Notation

This index is intended to aid cross-referencing, so notation that is specific to a single chapter is generally not listed. Some symbols are used locally, without ambiguity, in senses other than those given below; numbers following an entry are page numbers.

Unless otherwise stated, functions are real-valued and binary operations between functions such as $f \pm g$, $f \cdot g$, $f \wedge g$, $f \vee g$, comparisons $f \leq g$, $f < g$ or limiting relations $f_n \xrightarrow{n \rightarrow \infty} f$, $\lim_n f_n$, $\liminf_n f_n$, $\limsup_n f_n$, $\sup_n f_n$ or $\inf_n f_n$ are understood pointwise.

General notation: analysis

positive	always in the sense ≥ 0
negative	always in the sense ≤ 0
\mathbb{N}	$1, 2, 3, \dots$
$\inf \emptyset$	$\inf \emptyset = +\infty$
$a \vee b$	maximum of a and b
$a \wedge b$	minimum of a and b
$[x]$	largest integer $n \leq x$
$ x $	norm in \mathbb{R}^d : $ x ^2 = x_1^2 + \dots + x_d^2$
$x \cdot y$	scalar product in \mathbb{R}^d : $\sum_{j=1}^d x_j y_j$
$\mathbb{1}_A$	$\mathbb{1}_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$
δ_x	point mass at x
\mathcal{D}	domain
Δ	Laplace operator
∂_j	partial derivative $\frac{\partial}{\partial x_j}$
∇, ∇_x	gradient $(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_d})^\top$
$\mathcal{F}f, \hat{f}$	Fourier transform $(2\pi)^{-d} \int e^{-ix \cdot \xi} f(x) dx$
$\mathcal{F}^{-1}f, \check{f}$	inverse Fourier transform

$$\int e^{ix \cdot \xi} f(x) dx$$

$$e_\xi(x) \quad e^{-ix \cdot \xi}$$

General notation: probability

\sim	'is distributed as'
\perp	'is stochastically independent'
a.s.	almost surely (w. r. t. \mathbb{P})
iid	independent and identically distributed
$\mathbb{N}, \text{Exp}, \text{Poi}$	normal, exponential, Poisson distribution
\mathbb{P}, \mathbb{E}	probability, expectation
\mathbb{V}, Cov	variance, covariance
(L0)–(L3)	definition of a Lévy process, 7
(L2')	13

Sets and σ -algebras

A^c	complement of the set A
\overline{A}	closure of the set A
$A \cup B$	disjoint union, i.e., $A \cup B$ for disjoint sets $A \cap B = \emptyset$
$B_r(x)$	open ball, centre x , radius r

$\text{supp } f$	support, $\overline{\{f \neq 0\}}$
$\mathcal{B}(E)$	Borel sets of E
\mathcal{F}_t^X	canonical filtration $\sigma(X_s : s \leq t)$
\mathcal{F}_∞	$\sigma\left(\bigcup_{t \geq 0} \mathcal{F}_t\right)$
\mathcal{F}_τ	88
$\mathcal{F}_{\tau+}$	32
\mathcal{P}	predictable σ -algebra, 119

Stochastic processes

$\mathbb{P}^x, \mathbb{E}^x$	law and mean of a Markov process starting at x , 28
X_{t-}	left limit $\lim_{s \uparrow t} X_s$
ΔX_t	jump at time t : $X_t - X_{t-}$
σ, τ	stopping times: $\{\sigma \leq t\} \in \mathcal{F}_t, t \geq 0$
τ_r^x, τ_r	$\inf\{t > 0 : X_t - X_0 \geq r\}$, first exit time from the open ball $B_r(x)$ centered at $x = X_0$
càdlàg	right continuous on $[0, \infty)$ with finite left limits on $(0, \infty)$

Spaces of functions

$\mathcal{B}(E)$	Borel functions on E
$\mathcal{B}_b(E)$	— —, bounded
$\mathcal{C}(E)$	continuous functions on E
$\mathcal{C}_b(E)$	— —, bounded
$\mathcal{C}_\infty(E)$	— —, $\lim_{ x \rightarrow \infty} f(x) = 0$
$\mathcal{C}_c(E)$	— —, compact support
$\mathcal{C}^n(E)$	n times continuously diff'ble functions on E
$\mathcal{C}_b^n(E)$	— —, bounded (with all derivatives)
$\mathcal{C}_\infty^n(E)$	— —, 0 at infinity (with all derivatives)
$\mathcal{C}_c^n(E)$	— —, compact support
$L^p(E, \mu), L^p(\mu), L^p(E)$	L^p space w. r. t. the measure space (E, \mathcal{A}, μ)
$\mathcal{S}(\mathbb{R}^d)$	rapidly decreasing smooth functions on \mathbb{R}^d , 41